ABSTRACT

Given an arbitrary field F, two natural questions immediately arise. First, which polynomials in F[X] have roots in F? And second, which polynomials in F[X] are irreducible there? Clearly these questions are related, and many mathematicians will have intuitions about which of the two questions is easier -- but their intuitions do not always coincide.

The goal of this talk is to investigate this situation, and thereby to illustrate how computability theorists use their tools to resolve such questions. We will describe a standard method of comparison known as Turing reducibility, under which it turns out that the two problems are equivalent: given a solution to either one, we can produce a solution to the other as well. However, our second method of comparison, m-reducibility, yields finer distinctions, and using this we will give a definitive answer: one of these two problems actually is quantifiably easier than the other. To find out which one it is, come to the talk!