Implicit Function Calculation

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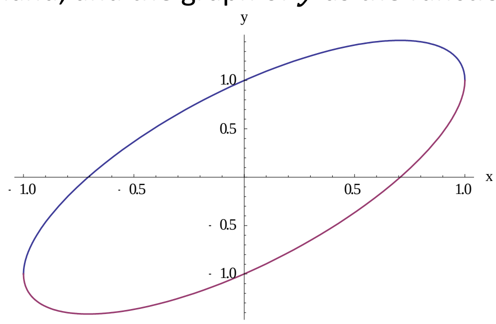
Many applied problems in Mechanics, Physics, and Mathematics can be formulated in the form of an equation $\boldsymbol{f(x,y)=0}$ that ties $\boldsymbol{x}$ and $\boldsymbol{y}$ together and in such a way defines a dependent variable $\boldsymbol{y}$ as the implicit function of the independent variable named $\boldsymbol{x}$**.** The role of the real-valued variable $\boldsymbol{x}$ usually plays one of the parameters describing the properties of the system under consideration; and the value of $\boldsymbol{y}$ can be assumed to be a real number as well. In Mathematics problems of this kind usually arise if you wish to solve an exact differential equation and then represent its solution explicitly. For example, while solving an exact differential equation, it is possible to come to an algebraic equation

$$x^2 + y^2 - 2\,x\,y - 1 = 0$$

that is very easy to solve by hand, and the graph of $\boldsymbol{y}$ as the function of $\boldsymbol{x}$ is a declined ellipse**:**



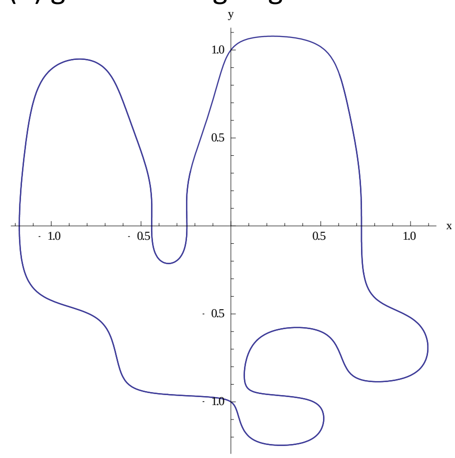
In more difficult case it is possible to get an equation like this one:

$$x^2 + y^2 - 1 - \cos\,(2y - 3y^2)\,\sin 5\,x = 0, \qquad (1)$$

that looks like a tough computational problem to solve with a computer only because there is no way to solve this equation for $\boldsymbol{y}$ by hand.

The problem of implicit function calculation is a fundamental problem of Applied Mathematics, but surprisingly our research revealed the absence of handy computer programs of good quality capable to calculate an implicit function in an easy and convenient way.

In order to resolve this problem, the author developed a simple theory and a Mathematica code that solves the problem of implicit function calculation within several seconds with a computer in many cases and provides a parametric representation of the solution in in the form $\boldsymbol{x = X(s)}$, $\boldsymbol{y = Y(s)}$**,** where the functions $\boldsymbol{X}$ and $\boldsymbol{Y}$ are provided by the Mathematica code, and $\boldsymbol{s}$ is the length of the arc along the solution curve on $\boldsymbol{x-y}$ plane. This natural representation of the solution is suitable to numerically approximate the values of the functions $\boldsymbol{X}(\boldsymbol{s})$ and $\boldsymbol{Y}(\boldsymbol{s})$ with the accuracy sufficient for many cases (including the derivatives - $\boldsymbol{X}'(\boldsymbol{s})$ and $\boldsymbol{Y}'(\boldsymbol{s})$ as well). For example, our calculation of the graph corresponding to the solution $\boldsymbol{y}(\boldsymbol{x})$ to the equation (1) gives a strange figure shown below:



It looks like an ameba. At the upcoming meeting the author is going to present the theory of the method, the algorithm, and the Mathematica code he created and demonstrate the power of the code. For verification purposes, the upcoming demonstration will include mostly solving equations of classical Mathematics, for which the solution curves are known and beautiful figures on $\boldsymbol{x}-\boldsymbol{y}$ plane. Besides, several new solutions that were found by means of the developed computer code will be demonstrated as well.