University of Puerto Rico Mayaguez Campus
Faculty of Arts and Sciences
Department of Mathematical Sciences
call box 9000, Mayagüez, P.R. 00681

THE DEPARTMENT OF MATHEMATICAL SCIENCES PROUDLY PRESENTS

COLLOQUIUM

SPRING 2013

The Mathematics Behind Feedback Control System

Dr. Gerson Beauchamp Báez and
Carlos G. Bolívar Vincenty

Electrical and Computer Engineering Department
University of Puerto Rico at Mayagüez

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ABSTRACT

The objective of this presentation is to show the role of mathematics on the problems related to control systems in electrical engineering. We present examples of dynamical systems for which a significantly deep mathematical analysis and design tools are required to accomplish the objective of effectively controlling the system. For example, we will present the Ball-and-Beam system and will develop its mathematical model from both Newtonian mechanics and Euler-Lagrange equations. Then the nonlinear equations are linearized through Jacobian Linearization, a state-feedback controller is designed by two methods: pole placement and Linear Quadratic Regulator (LQR), and simulations of the system performance are presented. Similarly, the modeling, design, control, and simulation of the simple pendulum as well as those for the inverted pendulum and cart system will be presented.

To apply Newtonian mechanics we use vector analysis. To develop Euler-Lagrange equations of motion, we encounter partial derivatives that are quite complicated to solve by hand if the system is not too simple. Sometimes we may need a symbolic mathematics tool such as Matlab's Symbolic Toolbox to handle these partial derivatives. To linearize the nonlinear equations of motion we use Jacobians for approximate linearization and, when possible, we could use exact feedback linearization. Once the equations of motion are linearized we need to perform a stability analysis to determine if the system is or not stable. Usually, the systems of interest are not stable or their dynamical behavior is not satisfactory. In that case, a feedback control system is used to stabilize the system and improve its dynamical behavior. Stability of the system is related to the eigenvalues of its *A* matrix. Knowledge of complex numbers and complex variables is required for stability analysis. State-variable feedback design is done through several manual and numerical algorithms that use matrix operations to determine the required feedback gains.

Monzón Building, Room 201, 10:45 AM
Refreshments will be served
15 minutes before the colloquium, M203