ACTIVITY #2 – CYLINDERS IN THREE DIMENSIONS

Name:______________________________

1. Draw each one of the following sets in three-dimensional Cartesian space.

   a. $\{(3, y, 0) : y \text{ is real}\}$
   b. $\{(3, y, z) : y, z \text{ reals}\}$
   c. How does this compare with the graph in 3D space of $x = 3$?
   d. $\{(0, y, z) : y + z = 4\}$
   e. $\{(x, y, z) : y + z = 4\}$
   f. How does this compare with the graph in 3D space of $y + z = 4$?

The following problems deal with the graph in three-dimensional space of equations where only two variables appear.

2. The graph of equation $z = x^2$ in three-dimensional space is a surface that consists of *all* points (x, y, z) in space that satisfy the equation. So, for example, point $(2, 3, 4)$ is on the graph of $z = x^2$ since its *z* coordinate is the square of its *x* coordinate, that is, $4 = 2^2$ (it does not matter what the value of *y* is).

   a. Find the (x, y, z) coordinates of 5 points that are on the graph of $z = x^2$ in three-dimensional space.
   b. Find the *z* coordinate of the following points if they are on the graph of $z = x^2$.
      $(-2,0,\quad)$ $(-1,0,\quad)$ $(0,0,\quad)$ $(1,0,\quad)$ $(2,0,\quad)$
   c. Draw the 5 points of part b in three-dimensional space. Observe that all of the points should be on the plane $y = 0$.
   d. How will the collection of *all* points of the form $(x, 0, z)$ that satisfy the equation $z = x^2$ look? Draw it.
   e. Find 5 points that are on the intersection of plane $y = 2$ with the graph of $z = x^2$ in three-dimensional space and represent them in a drawing.
   f. Draw the intersection of the plane $y = 2$ with the graph of $z = x^2$ in three-dimensional space. There is no need to draw the plane nor the surface, only the points that satisfy both conditions.
   g. Draw the intersection of the plane $y = -2$ with the graph of $z = x^2$ in three-dimensional space.
   h. How will the collection of *all* points (x, y, z) that satisfy the equation $z = x^2$ look? Draw it in three-dimensional space.

3. In this problem the graph of $y = |x|$ will be drawn in three-dimensional space. The graph of $y = |x|$ consists of all points in the set $\{(x, y, z) : y = |x|\}$. To do so:

   a. Draw the intersection of the plane $z = 0$ with the graph of $y = |x|$ in three-dimensional space.
   b. Draw the intersection of the plane $z = 1$ with the graph of $y = |x|$ in three-dimensional space.
   c. Draw the intersection of the plane $z = 2$ with the graph of $y = |x|$ in three-dimensional space.
   d. What happens as *z* is given more and more positive and negative values?
   e. Draw the graph of $y = |x|$ in three-dimensional space.

4. In this problem the graph of $y^2+z^2=1$ will be drawn in three-dimensional space. The graph of $y^2+z^2=1$ consists of all the points in the set $\{(x,y,z): y^2+z^2=1\}$.
   a. Draw the intersection of the plane $x=0$ with the graph of $y^2+z^2=1$ in three-dimensional space. Recall the equation of a circle of radius *r* with center at the origin.
   b. Draw the intersection of the plane $x=1$ with the graph of $y^2+z^2=1$ in three-dimensional space.
   c. Draw the intersection of the plane $x=-1$ with the graph of $y^2+z^2=1$ in three-dimensional space.
   d. What happens as *x* is given more and more positive and negative values?
   e. Draw the graph of $y^2+z^2=1$ in three-dimensional space.

5. In this problem the graph of $x=9-z^2$ will be drawn in three-dimensional space. The graph of $x=9-z^2$ consists of all points in the set $\{(x,y,z): x=9-z^2\}$.
   a. Draw in three-dimensional space all points where the plane $y=0$ intersects the graph of $x=9-z^2$.
   b. Draw in three-dimensional space all points where the plane $y=1$ intersects the graph of $x=9-z^2$.
   c. Draw in three-dimensional space all points where the plane $y=-1$ intersects the graph of $x=9-z^2$.
   d. What happens as *y* is given more and more positive and negative values?
   e. Draw the graph of $x=9-z^2$ in three-dimensional space.

6. Reflect on what was done in problems 3 to 5. How, in general, is the graph of an equation where only two variables appear drawn in three-dimensional space?

7. Let $S=\{(x,y,z): y=-z^2\}$.
   a. Find the intersection of the plane $y=-1$ with *S* and draw it in three-dimensional space without first drawing the graph of the entire surface. Observation: your drawing must be contained within the plane $y=-1$.
   b. Draw the intersection of the plane $z=1$ with *S* in 3D without first drawing the graph of *S*.
   c. Draw the intersection of plane $x=1$ with *S* in 3D without first drawing the graph of *S*.
   d. Draw the graph of $y=-z^2$ in three-dimensional space and verify that your graph is consistent with the answers to parts a, b, c.

8. In each of the following cases consider the set of all points in three-dimensional space that satisfy the given equation. Draw it in three-dimensional space.
   a. $x=2z$ b. $y=\sin(z)$ c. $x^2-y^2=1$ (recall conic sections)