ACTIVITY #4 – SECTIONS AND GRAPHS OF SURFACES

Name:____________________________________

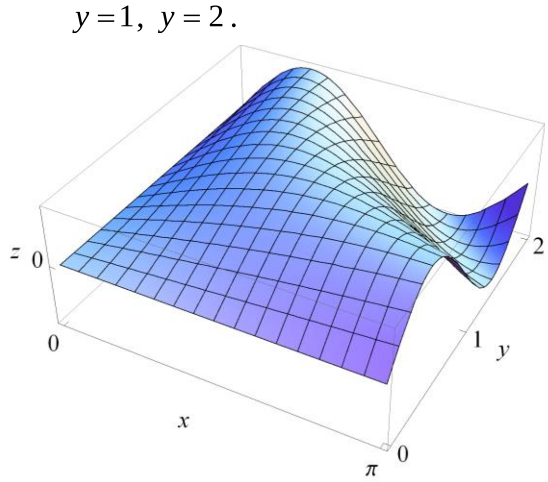
1. Let $f(x,y)=y^2$. The graph of *f* is the graph of $z=y^2$ in 3D space. To draw a graph using sections, first one chooses a variable to give values to. In this case, the easiest variable to use is the variable that is missing: one gives values to *x*.
   a. To understand the intersection of plane $x=2$ with the graph of $z=y^2$ complete the following table and represent the points in 3D space. Observe that all points must be on the plane $x=2$.

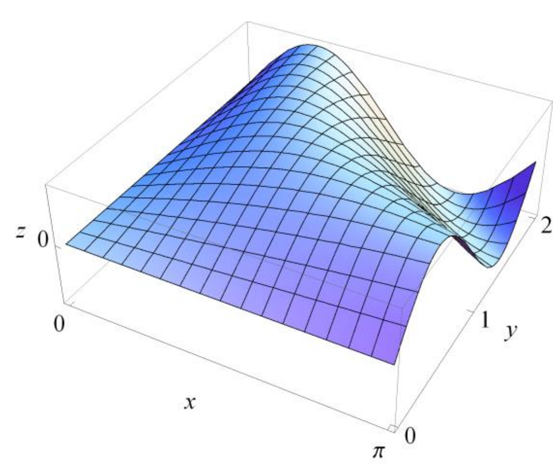
(x,y)	$(2,-2)$	$(2,-1)$	$(2,0)$	$(2,1)$	$(2,2)$
$f(x,y)$					

   b. Draw in 3D space the set of **all** points in the intersection of the plane $x=2$ with the graph of *f*.
   c. Draw in 3D space the intersection of the graph of *f* with each one of the planes $x=-1$, $x=0$, $x=1$.
   d. Draw the graph of *f*.

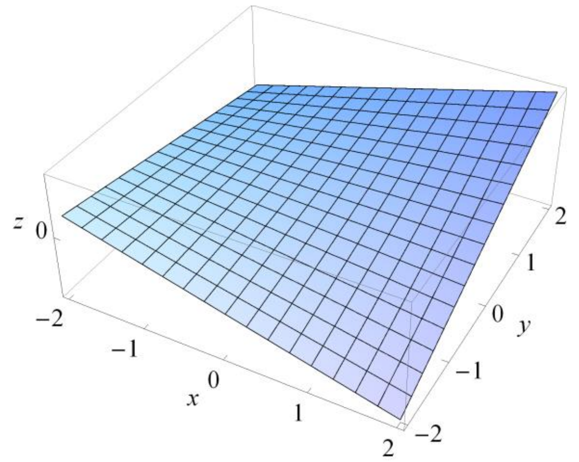
2. In this problem the graph of the function $f(x,y)=x^2+y$ will be drawn using sections. To do so, follow the instructions:
   a. Start by observing that if values are given to the variable *y* then you'll obtain quadratic functions whose graph you should be able to draw with no difficulty. Proceed to do this: draw the intersection of the set $S=\{(x,y,z): z=x^2+y\}$ with each one of the planes $y=0$, $y=1$, $y=2$.
   b. What kind of curves are obtained as you give *y* bigger and bigger posite values? And what if you give *y* negative values of bigger and bigger magnitude?
   c. Now one may give a value to another variable so that the resulting curve may be used as a framework upon which to place the other curves that have previously been obtained. Draw the intersection of the set $S=\{(x,y,z): z=x^2+y\}$ with the plane $x=0$.
   d. Consider the answers to parts a,b, c to conclude how the graph looks and draw it.

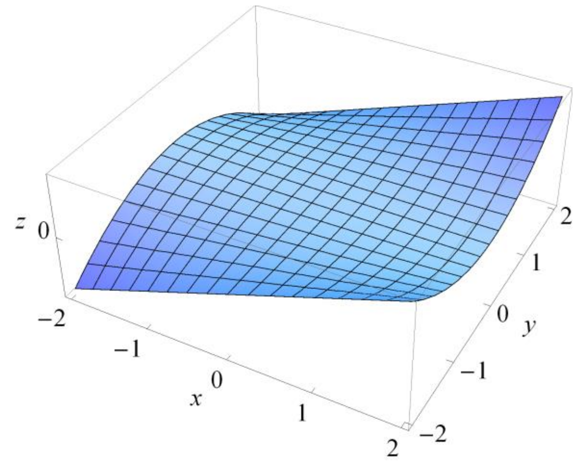
3. The following are two copies of the graph of $z=f(x,y)$. On the left hand copy darken and identify the points where the surface intersects the planes $x=0$, $x=\pi/2$, $x=\pi$. On the right hand copy darken sand identify the points where the surface intersects the planes $y=0$, $y=1$, $y=2$.

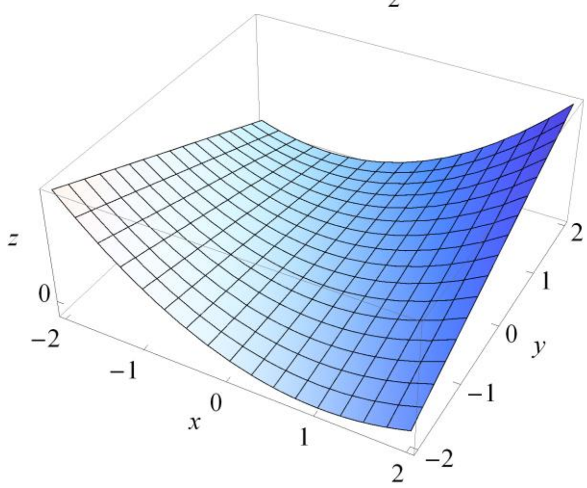


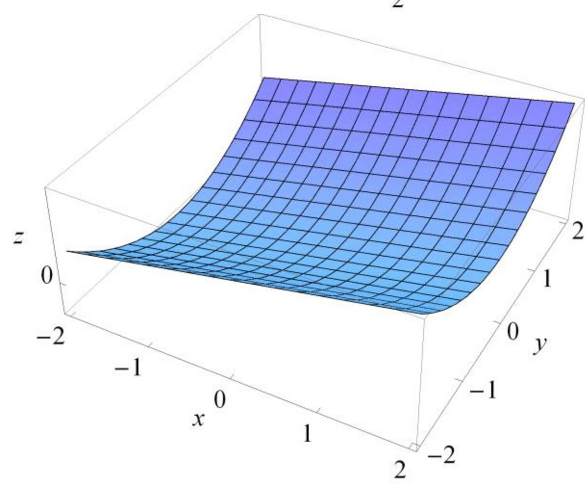


4. Write the corresponding formula besides each one of the following graphs. Choose between: $z = xy^2$, $z = x + y^2$, $z = xy + y$, $z = xy + x^2$. **Use sections to completely justify your answer.**

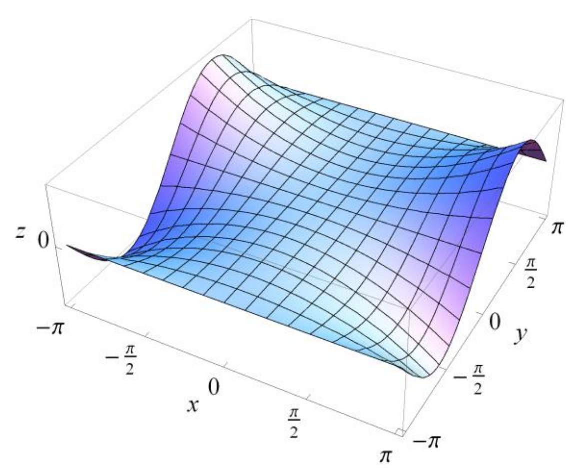


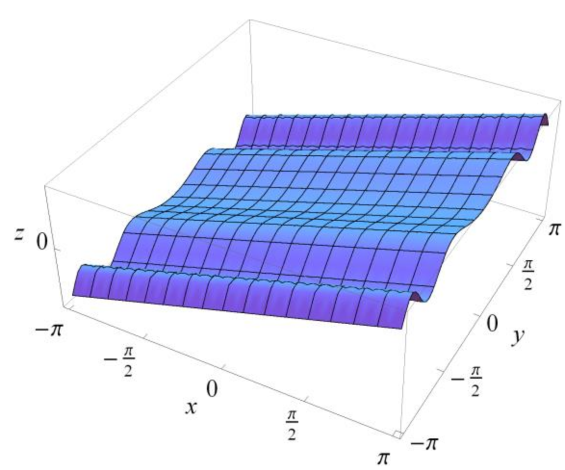


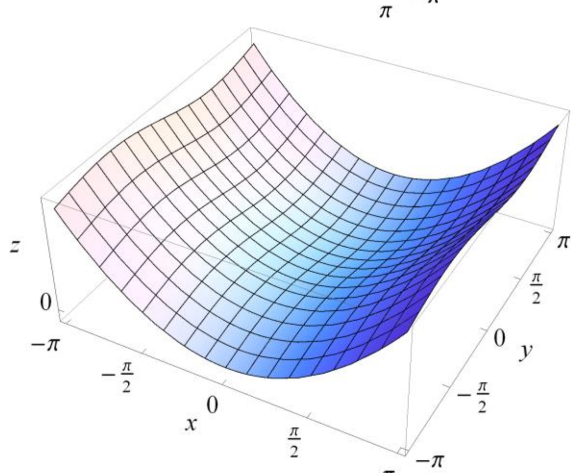


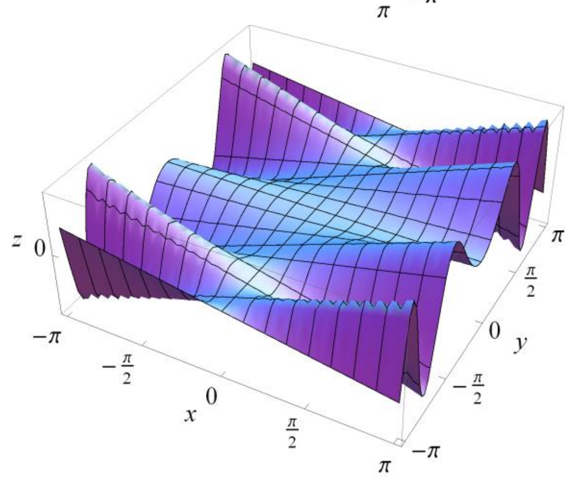


5. Write the corresponding formula besides each one of the following graphs. Choose between: $z = x^2 \sin(y)$, $z = x^2 - \sin(y)$, $z = x\sin(y^2)$, $z = x + \sin(y^2)$. **Use sections to completely justify your answer.**

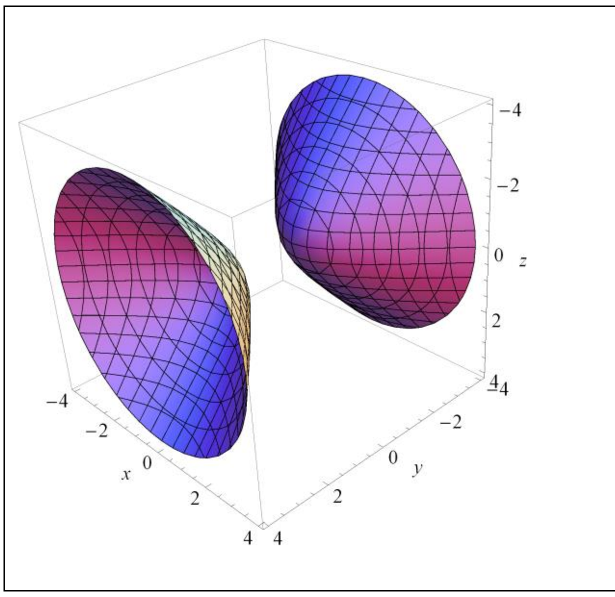


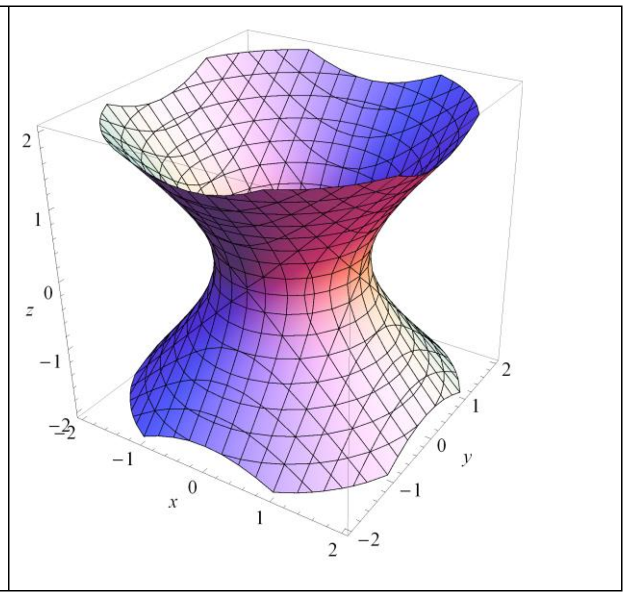






6. Consider the surface with equation $x^2 - y^2 + z^2 = -1$.
   a. To what variables may one give values in order to obtain circles?
   b. Draw in 3D space the intersection of the planes $y = 1$ y $y = -1$ with the surface.
   c. What happens if one substitutes y for a value $-1 < y < 1$? What does this mean in terms of the graph of the surface?
   d. What happens if one gives y values $y > 1$? $y < -1$? What does this mean in terms of the graph of the surface?
   e. Now give values to other variables in order to obtain a framework. Draw in 3D space the intersection of plane $x = 0$ with the surface (observe that the equation of a conic section is obtained).
   f. Draw in 3D space the intersection of the plane $z = 0$ with the surface (observe that the equation of a conic section is obtained).
   g. Identify the graph of $x^2 - y^2 + z^2 = -1$ between the two graphs given below. Make sure that your answers to the previous parts of the question are consistent with the graph you choose.





7. Let $f(x, y) = yx^2$ with domain restricted to $\{(x, y) : -1 \le x \le 1, -1 \le y \le 1\}$. The graph will now be drawn.
   a. Draw the transversal section corresponding to $y = -1$ in the space provided.
   b. Draw the transversal section corresponding to $y = 1$ in the space provided.
   c. Draw the transversal section corresponding to $x = -1$ in the space provided.
   d. Draw the transversal section corresponding to $x = 1$ in the space provided.
   e. **Use other transversal sections** as needed to complete the graph of the function.