

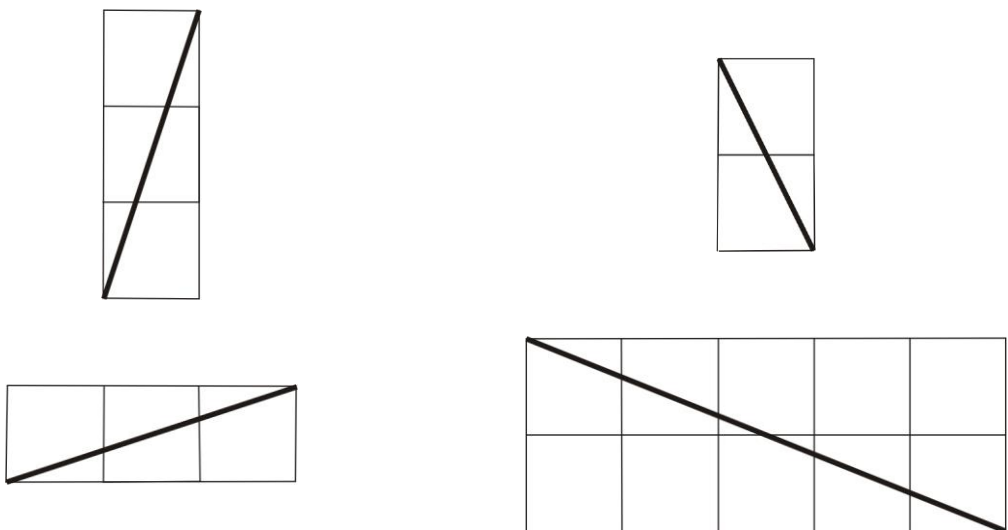
ACTIVITY #6 – SLOPE IN TWO AND THREE DIMENSIONS AND VERTICAL CHANGE  
ON A PLANE

Name: \_\_\_\_\_

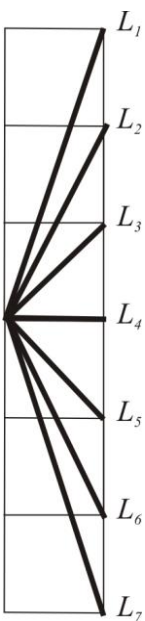
VERTICAL CHANGE AND SLOPE IN TWO DIMENSIONS

The first three problems explain how to compute slope looking at a figure, **WITHOUT USING FORMULAS**; visually one may determine the sign (increasing-positive, decreasing-negative) of the slope and the magnitudes of vertical and horizontal changes may be determined by counting.

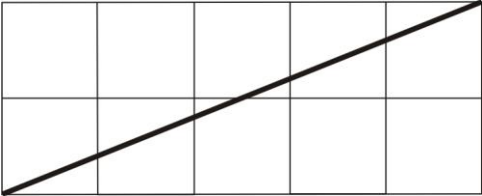

1. In the following figures, each square has the same vertical and horizontal measure of one unit. In each case:
- Determine the sign of the slope of the line.
  - Choose two points on the line and find the magnitudes of the vertical and horizontal change from one point to the other (**WITHOUT USING FORMULAS**).
  - Find the slope of the line (**WITHOUT USING FORMULAS**).



- 2a. **WITHOUT USING FORMULAS**, find the slope of each one of the following lines (each square measures one unit vertically and horizontally).
- b. What is the vertical change in  $L_1$  for a horizontal change of 2 units?
- c. What is the vertical change in  $L_2$  for a horizontal change of 5 units?
- d. Explain in your own words how the vertical change on a line may be found if the slope and horizontal change are given.



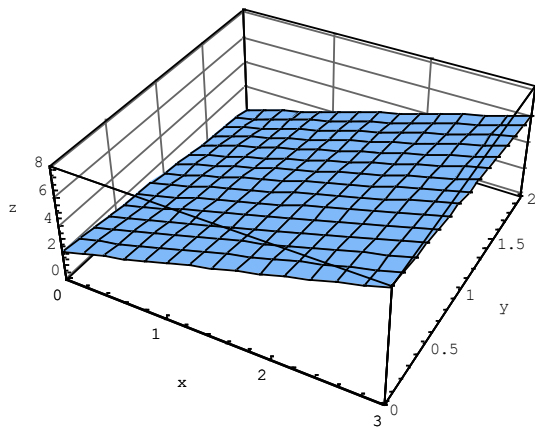
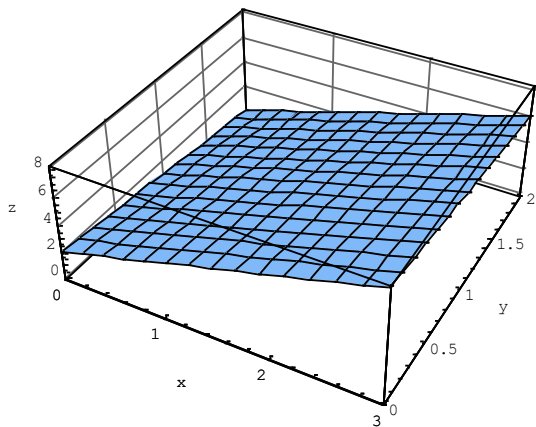
3. In each one of the following lines, find the vertical change that corresponds to each given horizontal change (Use the observation of problem 2d to find the exact value).

| Line  | Horizontal change | Vertical change |
|---|-------------------|-----------------|
|  | $\Delta H = 3$    | $\Delta V =$    |
|   | $\Delta H = 15$   | $\Delta V =$    |
|  | $\Delta H = 3$    | $\Delta V =$    |
|   | $\Delta H = 15$   | $\Delta V =$    |

### SLOPES IN THE X AND Y DIRECTIONS OF A PLANE

A non-vertical line in space is said to be **in the  $x$  direction** if it is in a plane of the form  $y = c$  for some constant  $c$ .

4. On the plane that is below on the left, darken three lines that are in the  $x$  direction.



A non-vertical line in space is said to be **in the  $y$  direction** if it is in a plane of the form  $x = c$  for some constant  $c$ .

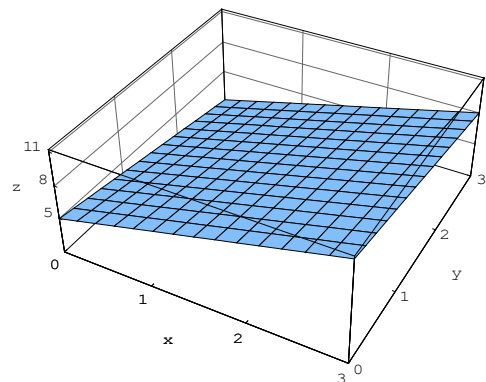
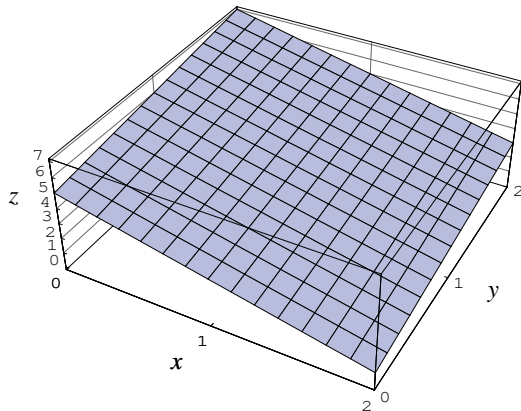
5. On the plane that is above to the right, darken three lines that are in the  $y$  direction.

The **slope of a line in the  $x$  direction** is computed like slopes of lines in two dimensions (like in problems 1, 2, and 3), keeping in mind that in three dimensions *vertical* means up or down, that is, in the  $z$  direction:

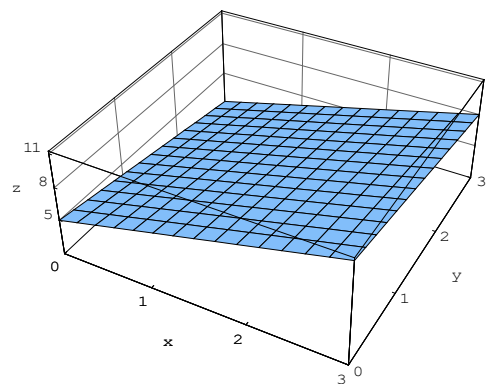
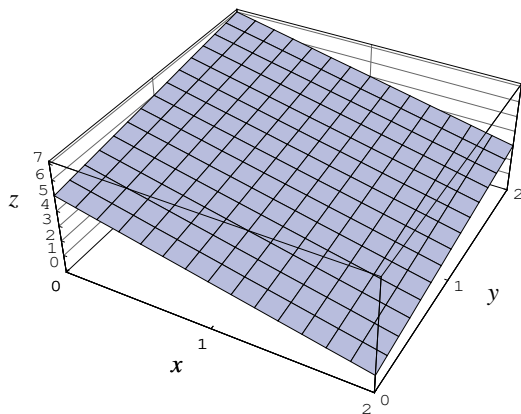
- Decide the sign: if when  $x$  increases  $z$  increases then the sign is positive; if when  $x$  increases  $z$  decreases then the sign is negative.
- Take two points that are on the line in the  $x$  direction.
- The magnitude of the slope is equal to the vertical change  $dz$  from one point to the other over the horizontal change  $dx$ , from one point to the other.

The **slope of a line in the  $y$  direction** is defined in a similar manner; vertical change over horizontal change,  $dz / dy$  , with the sign depending on whether  $z$  increases (positive) or decreases (negative) when  $y$  increases.

6. In each of the following planes identify a line in the  $x$  direction and find its slope (First read the instructions that precede this exercise and then choose a line with two points with coordinates that may be determined easily).



7. In each of the following planes identify a line in the  $y$  direction and find its slope (choose a line with two points with coordinates that may be determined easily).



8. Let  $P$  be the plane with equation  $z = 2x + 3y + 4$ .
- The intersection of  $P$  with the fundamental plane  $x = 5$  is a line with slope  $m = ?$
  - Reflect on what you did in the previous part (problem 8b) and explain why in general all the lines on a plane, that are in the  $y$  direction (this means that  $x$  is constant on the line) have the same slope.
  - The intersection of  $P$  with the fundamental plane  $y = 3$  is a line with slope  $m = ?$
  - Reflect on what you did in the previous part (problem 8c) and explain why in general all the lines on a plane, that are in the  $x$  direction (this means that  $y$  is constant on the line) have the same slope.

The previous problem suggests that on a plane, all lines in the  $x$  direction have the same slope (are parallel); this is called  $m_x$ , the **slope in the  $x$  direction of the plane**.

Similarly, in a plane all lines in the  $y$  direction have the same slope which is called  $m_y$ , the **slope in the  $y$  direction of the plane**.

9. A plane has the following table. The entries in the table are the  $z$  values for the given  $x$  and  $y$  values.
- Find the slope in the  $x$  direction of the plane.
  - Briefly explain why the same slope will be obtained regardless of what two points are used to compute it.
  - Find the slope in the  $y$  direction of the plane.

| $x / y$ | 2  | 4 | 6 |
|---------|----|---|---|
| 1       | 5  | 3 | 1 |
| 2       | 8  | 6 | 4 |
| 3       | 11 | 9 | 7 |