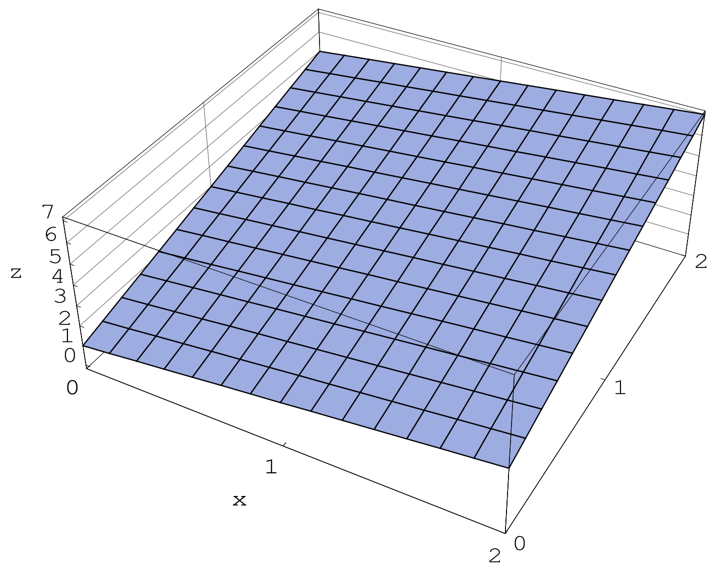
ACTIVITY #7 – VERTICAL CHANGE AND THE EQUATION OF A PLANE

Name:____

VERTICAL CHANGE ON A PLANE

1. This problem refers to the plane in the following figure. Recall that on a line:
$$(\text{vertical change}) = (\text{slope}) \times (\text{horizontal change})$$



a. If you start at point (0,0,1) of the above plane and move on the plane in the *x* direction in such a way that *dx* (the horizontal change in *x*) is 2, find dz_x (the vertical change in the *x* direction). On the above graph, darken and label a **horizontal** segment that represents *dx* (since the segment is horizontal it will NOT be on the plane) and a vertical segment that represents dz_x. If you now continue moving on the plane, but this time in the *y* direction in such a way that *dy* (the horizontal change in *y*) is 2, find dz_y (the vertical change in the *y* direction). On the above graph, darken and label a horizontal segment that represents *dy* (since the segment is horizontal, it is NOT on the plane) and a vertical segment that represents dz_y.

b. Use the fact that on a line, the vertical change is the slope multiplied by the horizontal change to complete the following table, one line at a time. The spaces labeled with * are filled with numbers and the ones labelled with ** are filled with an expression in the variables *dx*, *dy*.

Initial point	Horizontal change in the *x* direction dx	Vertical change in the *x* direction dz_x	Horizontal change in the *y* direction dy	Vertical change in the *y* direction dz_y	Total vertical change $dz = dz_x + dz_y$
$(0,0,1)$	3	*	5	*	*
$(3,4,12)$	4	*	2	*	*
(a,b,c)	dx	**	dy	**	**

c. Explain in your own words why the formula $dz = dz_x + dz_y$ makes sense.

d. Explain in your own words why on a plane it is true that $dz = m_x dx + m_y dy$.

e. Reflect on what you did on the previous parts and then explain in your own words how the vertical change from one point on the plane to another point on the plane may be found if you don't know the *z* coordinate of the final point but you know the change in *x*, the change in *y*, and the slopes in the *x* and *y* directions.

f. Explain in your own words why the expression for total vertical change in the third row of the above table does not depend on the initial point (*a*, *b*, *c*).

2. The following is a table of values of a plane:

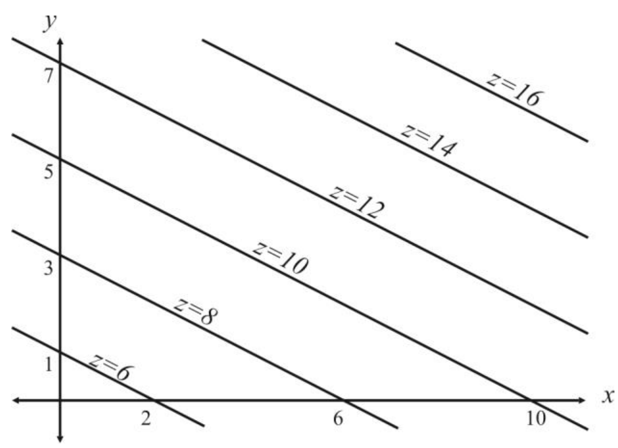
y / x	2	4	6
1	**5**	**3**	**1**
2	**8**	**6**	**4**
3	**11**	**9**	**7**

   a. Find m_x.

   b. Find m_y.

   c. Do as in part 1d of the previous problem to express vertical change *dz* as a function of the horizontal change in the *x* direction and the horizontal change in the *y* direction (that is, in terms of the variables *dx* and *dy*).

3. The following is a contour diagram of a plane.



   a. Find m_x (Remember that the plane is in 3D and that therefore both *x* and *y* refer to "horizontal", while "vertical" refers to *z*).

   b. Find m_y (Remember that the plane is in 3D and that therefore both *x* and *y* refer to "horizontal", while "vertical" refers to *z*).

   d. Express vertical change *dz* as a function of the horizontal change in the *x* direction and the horizontal change in the *y* direction (that is, in terms of *dx* and *dy*).

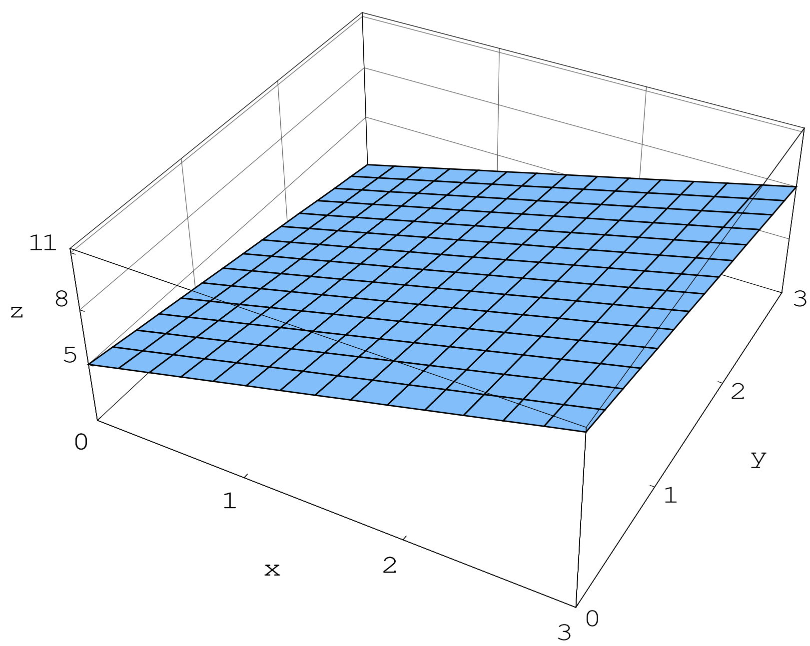
4. In the following table, each column corresponds to a plane. Different columns correspond to different planes. Proceed one column at a time to complete the missing information in the table. The spaces in column 1 are filled with numbers. Reflect on what you did in column 1 to fill column 2 with expressions in *x* and *y*. Reflect on what you did in column 2 to fill column 3 with expressions in *x* and *y* (x_0, y_0, m_x, m_y are treated as constants).

	Plane #1	Plane #2	Plane #3
Slope in the *x* direction, m_x	4	-1	m_x
Slope in the *y* direction, m_y	2	-2	m_y
Initial point	(3,-2,4)	(3,1,2)	(x_0, y_0, z_0)
Final point	(7,1,*z*)	(*x*,*y*,*z*)	(x, y, z)
Vertical change in the *x* direction, dz_x			
Vertical change in the *y* direction, dz_y			
Total vertical change, *dz*			
z coordinate of the final point			

EQUATION OF A PLANE

5. The equation of a plane is an equation where the only variables that may appear are *x*, *y*, *z* and where the points that satisfy the equation are precisely the points on the plane. Reflect on what you did on the second and third column of the previous problem to explain how the notion of total vertical change (*dz*) may be used to find the equation of a plane if you know a point on the plane and the slopes in the *x* and *y* directions.

6. Use the notion of vertical change on a plane *dz* to find the equation of the plane represented in the following figure:



7. Use the notion of vertical change on a plane *dz* to find the equation of the plane represented in the following table:

y *x*	0	3	6
2	18	20	22
4	15	17	19
6	12	14	16