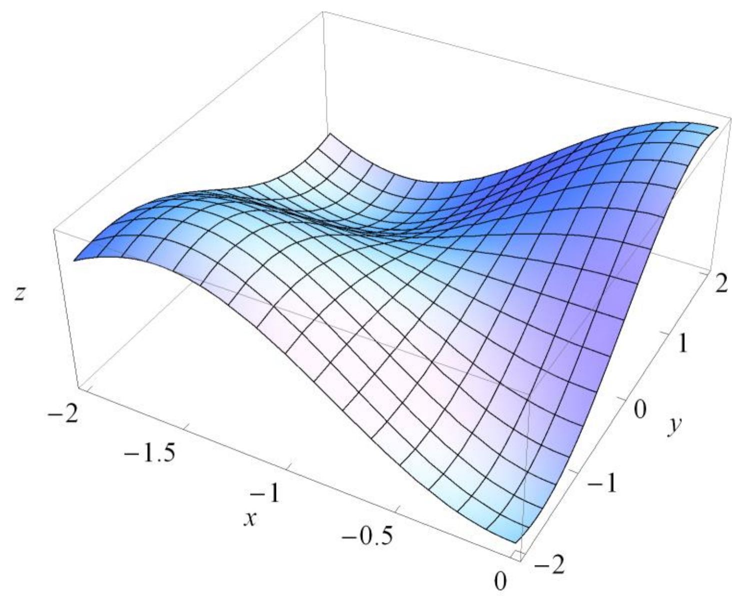
ACTIVITY #8 – PARTIAL DERIVATIVES AND TANGENT PLANE

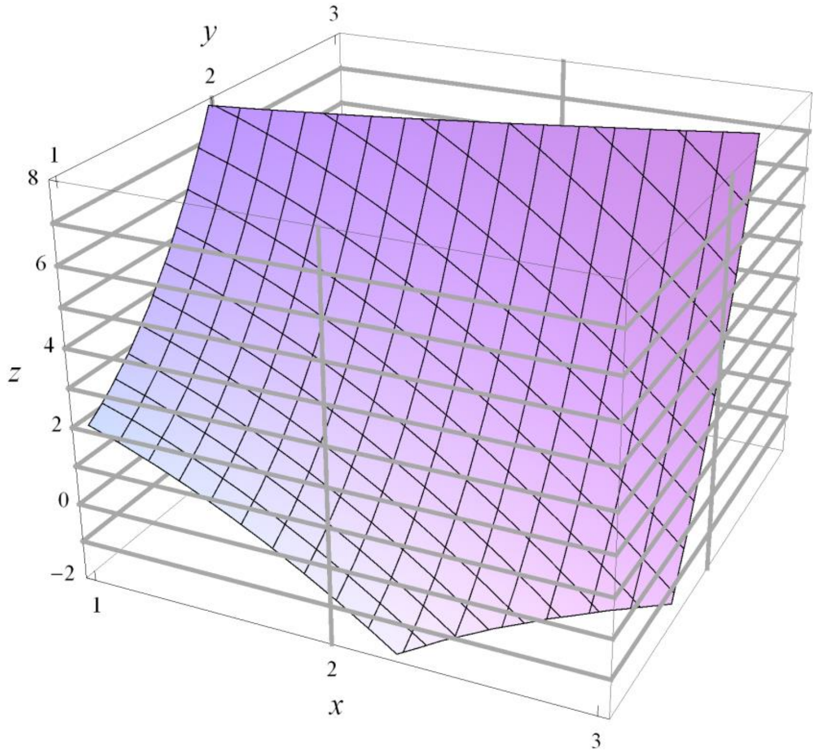
Name:____

A non-vertical line is said to be in **the *x* direction** when the *y* coordinate of its points is constant and is said to be **in the *y* direction** when the *x* coordinate of its points is constant.

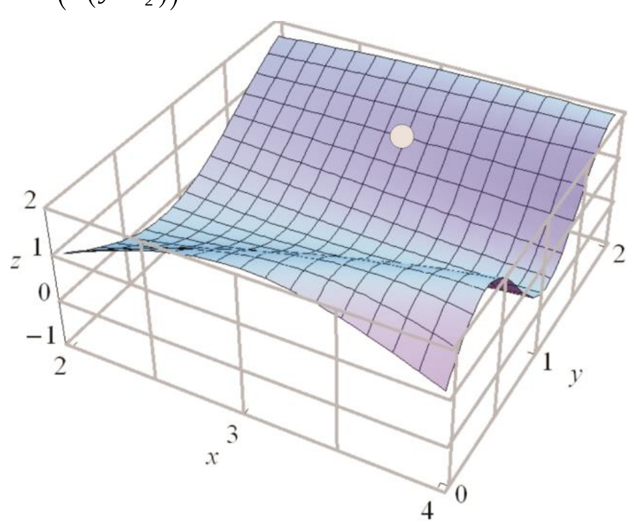
1. The following is the graph of $f(x,y)=\cos(2x)\sin(y)$.
   a. Locate the point $P(-\pi/2,-2,f(-\pi/2,-2))$ on the graph of *f*. (Suggestion: $\pi/2\approx 1.57$)
   b. Use calculus to find the slope of the line in the *x* direction that is tangent to the graph of *f* at point *P*. (Remember that a derivative is the slope of a tangent line. If you use the calculator, although it is not needed, remember to put it in "radian mode".)
   c. Draw the line in the *x* direction that is tangent to the graph of *f* at point *P*.
   d. Remember that a secant line at *P* in the *x* direction is a line that goes from *P* to a point $Q(-\pi/2+\Delta x,-2,f(-\pi/2+\Delta x,-2))$ on the graph. Draw the secant lines that correspond to horizontal changes of $\Delta x = 1.5,\ 0.5,\ 0.05$
   e. Find the slopes of the secant lines of the previous part and state what is the relation between the slopes of the secant lines and the slope of the tangent line.



2. The following is part of the graph of $f(x,y)=y^3-x^2+2$.
   a. Verify that the point $P(1,1,2)$ is on the graph of *f* (justify your answer) and darken the point on the graph.
   b. Find the slope of the line in the *x* direction that is tangent to the graph of *f* at the point $P(1,1,2)$ (use calculus). Draw the tangent line as carefully as possible for $1\le x\le 3$ (this may be done using point $P(1,1,2)$ and the slope). The line you draw should be on the plane $y=1$, tangent to the curve.
   c. Find the slope of the line in the *y* direction that is tangent to the graph of *f* at the point $P(1,1,2)$. Draw the tangent line as carefully as possible for $1\le y\le 3$.
   d. The tangent lines drawn in parts b and c are part of the tangent plane to the graph of *f* at the point $P(1,1,2)$. Finish drawing the tangent plane as carefully as possible in all the region with $1\le x\le 3$ and $1\le y\le 3$.
   e. Find the vertical change *dz* from the point $P(1,1,2)$ to a generic point (x,y,z) on the tangent plane and express it in terms of the variables *x* and *y*. How may *dz* be used to find the equation of the tangent plane?



3. The graph of $f(x,y) = \sin\left(x\left(y - \frac{\pi}{2}\right)\right) + 1$ is shown below.

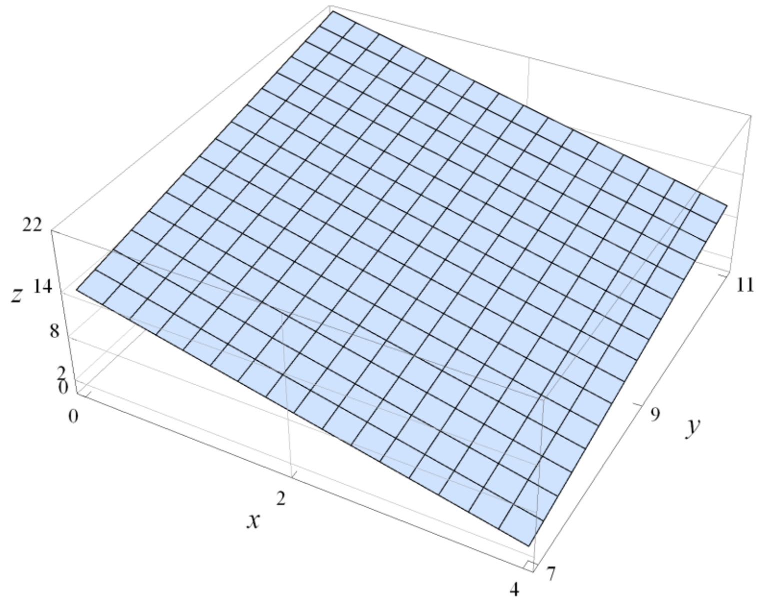


a. It is known that a tangent line is a limit of secant lines. Draw in the surface above two secant lines in the *y* direction that go through the point $(3, 1.5, f(3, 1.5))$ (the point highlighted on the graph) and the tangent line to the graph at the given point in the *y* direction. Find the slope of this tangent line.

b. Draw on the above surface the line in the *x* direction that is tangent to the graph at the point $(3, 1.5, f(3, 1.5))$ (highlighted on the graph). Find the slope of this tangent line.

c. The two tangent lines drawn in parts a and b are part of the tangent plane to the graph of the function at the point $(3, 1.5, f(3, 1.5))$. Find the equation of the tangent plane (You know a point and two slopes).

4. The following is the graph of the tangent plane to the graph of $z = f(x, y)$ at the point (2,7,8).
   a. Find $f_x(2,7)$ and $f_y(2,7)$.
   b. Find the equation of the tangent plane.



5. The following is a table of the tangent plane to the graph of $z = f(x, y)$ at the point (4,6,6).
   a. Find $f_x(4,6)$ and $f_y(4,6)$.
   b. Find the equation of the tangent plane.

y / x	3	6	9
2	5	2	−1
4	9	6	3
6	13	10	7

6. Find the equation of the tangent plane to the graph of $f(x, y) = \dfrac{1}{x+2y}$ at the point $\left(2, 1, f\left(2,1\right)\right)$.