

COMP 6785 assignment 1

1. Show that the relation between functions mapping $[0, \infty[$ to itself,

$$f \mathcal{R} g \text{ if } f \in \Theta(g),$$

is an equivalence relation (reflexive, symmetric, transitive). What can you say about the similar relations corresponding respectively to $f \in O(g), f \in o(g)$?

2. Certain algorithms (sorting is one example) which use divide and conquer lead to the recurrence

$$T(n) = T(\lfloor n/2 \rfloor) + T(\lceil n/2 \rceil) + n, \quad T(2) = T_0.$$

Solve directly this recurrence (find a suitable order of growth of $T(n)$) as we did in class. Do not use the master theorem.

- 3.

(a) Show that $\left\lceil \frac{n}{2^{h+1}} \right\rceil = O(n/2^h)$. Here, n and h are arbitrary integers.

(b) Show that an n -element heap has height $\lfloor \lg n \rfloor$. Hint: how many nodes are there in a full binary tree of height h ?

(c) Show that there are at most $\lceil n/2^{n+1} \rceil$ nodes of height h in an n -element heap.

4. Exercise C.2-2 of Cormen & al.
5. Exercise C.2-3.
6. Exercise C.2-4.