MATE 3021 Practice exam 1 September 24, 2014

"log" means natural logarithm. There are seven problems.

1. Find the values of x where the given function is continuous:

$$\text{(a) } f(x) = \log\frac{x}{1+x} \qquad \text{(b) } f(x) = \exp(-\sqrt{x-1}) .$$

(In each case, use a composition diagramme for the function:

$$x \mapsto \ldots \mapsto \ldots)$$

2. State the five limit laws for functions, numbering them. Specify exactly how you are using them to find the limit:

   (a) Limit of $\frac{(x-1)^2}{x^2-1}$ as $x \to 1$.

   (b) Limit of $2x^3 - 2x + 1$ as $x \to -2$.

3. Consider the function $x \mapsto \frac{1}{\sqrt{(1-x)(x+1)}}$. What symmetry does it have? Plot it. Using blue for its graph, green for its domain, red for its range, show graph, domain and range all on the same plot, and specify how domain and range are obtained from the graph, using projections.

4. Write a formula for $N(t)$ if $N(0) = 4$, N triples every forty minutes, and one unit of time is fifteen minutes.

5. State in recursive form $N(t) = 20 \cdot 3^t$, and plot $N(t)$ for $t = 0, 1, 2, 3, 4$.

6. Consider the sequence $a_n = 2\frac{(-1)^n}{n}$, $n \geq 1$. Plot the first few terms, where n is on the horizontal axis.

   (a) Find a radius r such that the region $-r \leq a_n \leq r$ contains all points of the sequence but one. Plot.

   (b) Find a radius r such that the region $-r \leq a_n \leq r$ contains all points of the sequence but one hundred. No need to plot.

   (c) If r is some positive number, is it the case that for some index $n(r)$ (simply indicating that n depends on r), the point (n, a_n) will have $-r \leq a_n \leq r$ for all $n \geq n(r)$? If yes, find $n(r)$, and explain what you conclude about the limit of a_n as $n \to \infty$.

7. Find all possible limits of a sequence which satisfies the recursion

$$a_n = 2a_{n-1}(1 - a_{n-1}) .$$