

“log” means natural logarithm. There are seven problems.

1. Find the values of x where the given function is continuous:

$$(a) f(x) = \log \frac{x}{1+x} \quad (b) f(x) = \exp(-\sqrt{x-1}).$$

(In each case, use a composition diagramme for the function:

$$x \mapsto \dots \mapsto \dots)$$

- 2.

(a) Simplify:

$$(i) \log_2 16^x \quad (ii) \log_{1/2} 8^{-x} \quad (iii) \exp(-2 \log x)$$

(b) Show that if $0 < a < 1$, the function $y = a^x$ can be written as $e^{-\mu x}$ where μ is a positive constant. Express μ in terms of a .

3. A sequence satisfies the recursion $a_n = \frac{5}{2 - a_{n-1}}$. Can this sequence have a limit? Explain.

4. Find the tangent line to the hyperbola $yx = c$, where c is a positive constant, at the point (x_1, y_1) with $x_1 > 0$. Show that the tangent line intersects the x -axis at a point that does not depend on c .

- 5.

(a) Show that the tangent line to the curve $y = x^2$ at the point $(1, 1)$ passes through the point $(0, -1)$.

(b) Find all points on the curve $y = 2x^3 - 4x + 1$ where the tangent line is parallel to the line $y - 2x = 1$.

6. Find the limit as $x \rightarrow -\infty$ of $\frac{\sin x}{x}$, explaining each step.

7. Find the limit:

$$(a) \lim_{x \rightarrow -\infty} \frac{3 + x^2}{1 - 2x^2}$$

$$(b) \lim_{x \rightarrow \infty} \exp(\exp(-x)).$$