1. Sketch the curve

$$y = \frac{\sin x}{1 + \cos x}.$$

Period, variation, inflection points, asymptotes, regions of convexity or concavity.

2. Find any asymptotes:

(a) $y = \dfrac{4x^3 - 2x^2 + 5}{2x^2 + x - 3}$

(b) $y = \dfrac{5x^4 + x^2 + x}{x^3 - x^2 + 2}$

3. A woman at a point A on the shore of a circular lake with radius 2 mi wants to arrive at the point C diametrically opposite A on the other side of the lake in the shortest possible time. She can walk at the rate of 4 mi/h and row a boat at 2 mi/h. How should she proceed?

4. Fermat's principle in optics states that light always travels from one point to another along a path that minimises the travel time. Consider light from a source point A, reflected by a plane mirror to a receiver point B. Show that for the light to obey Fermat's principle, the angle of incidence must equal the angle of reflection.

5. Find a function f such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of f.

6. The linear density of a rod of length 1 m is given by $\rho(x) = 1/\sqrt{x}$, in grams per centimeter, where x is measured in centimetres from on end of the rod. Find the mass of the rod.

7.

(a) Let A_n be the area of a polygon with n equal sides inscribed in a circle of radius r. By dividing the polygon into n congruent triangles with central angle $2\pi/n$, show that

$$A_n = \frac{1}{2} n r^2 \sin\left(\frac{2\pi}{n}\right)$$

(b) Show that $\lim_{n\to\infty} A_n = \pi r^2$.

8.

(a) Express $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$ as a definite integral.

(b) Find $\int_1^2 x^{-2}\,dx$. Hint: choose x_i^* to be the geometric mean of x_{i-1} and x_i (that is, $x_i^* = \sqrt{x_{i-1}x_i}$) and use the identity

$$\frac{1}{m(m+1)} = \frac{1}{m} - \frac{1}{m+1}.$$