Exam 1

1.b) This example was covered in class. We saw two ways of finding the limit: (i) by comparison with $y = x$ and (ii) by factoring. Other methods of proof tend to belong to the style known as handwaving.

2. We have learnt to identify and name this kind of curve. To sketch it, it suffices to find the location of the asymptotes and of the branches. For the latter, locating a single point on the curve is enough, but note that this can be replaced by finding the limit of $f(x)$ on each side of the vertical asymptote. For this, show a table of the sign of y on different x-intervals (not what *do not tabulate* refers to).

3.a) At this stage of the course, to find a slope, you must find the limit of a difference quotient (that is, use the definition of derivative). Two ways of writing the difference quotient are

$$\frac{y(-1+h)-y(-1)}{h} \quad \text{and} \quad \frac{y(x)-y(-1)}{x-(-1)}$$

They both allow cancelling common factors to numerator and denominator. Which one is easier?

4. The function is given by different formulæ on each of the intervals $(-\infty, 1)$, $[1, \infty)$. For continuity at $x = 1$, you must show that the one-sided limits agree with the value of the function at one. For continuity at all other points, you may use the fact: if two functions f and g agree on any *open* interval (bounded, semi-infinite or infinite), and g is continuous on that interval, then f is also continuous on that interval. Here, the suitable intervals are $(-\infty, 1)$ and $(1, \infty)$. Note that $[1, \infty)$ is not open.

5. In order to use the bisection method, cast the equation to solve in the form $f(x) = 0$. f has to be continuous, and, together with the initial interval $[a, b]$, obey the hypothesis of the intermediate value theorem.

Exam 2

3. The given function is periodic. If there is one such point, there are infinitely many.

4. Implicit differentiation.

5.a) The sine function is periodic. If the domain of f is bounded, indicate which finite interval $[a, b]$ contains it. If you can show that D, domain of f, contains a set which is itself not bounded, you will conclude that D is not bounded. The set in question does not have to be an interval, it may be a sequence of points.

7. What are the unknowns? A suitable choice is one number for the ordinate of the centre of the circle (say, $(0, b)$) and the point of contact of the circle and the parabola in the first quadrant, say, (x_0, y_0) (the other point of contact will be a symmetric point in the second quadrant). Since there are three unknowns, you will need three equations.

Exam 3

1. The reference to hypothenuse suggests that the triangle is a right triangle, but the correct text should read "one side of a right triangle… ". Keep in mind that the derivative formulæ of trigonometric functions assume that the unit is the radian. So to use them, you must convert

from degrees to radians.

3. $\frac{d}{dx}e^{2x} = ?$

5.

(a) For a complete answer, state which property of functions insures that the limit of the function as $x \to a$ is the value of the function at $x = a$.

(b) What does Rolle's theorem say?

7. In this kind of problem, you must provide symbols for a point on the curve (say, (x_0, e^{x_0}) or (a, e^a) since you need to find this point) and the generic point (say, (x, y)) on the tangent line.