Some common corrections

1. The following symbols have different meanings:

   (a) The symbol " $=$ " means *equals*, and nothing else. It connects expressions, sets, quantities, or any objects of the same nature, but not statements. It is not an all-purpose connector, nor does it open a paragraph.

   (b) The symbol " $\Rightarrow$ " means *implies*. It connects sentences or logical statements; it does not connect values or expressions. For instance:

   $$\sin^2 x + \cos^2 x = 1 \;\Rightarrow\; 2\cos^2 x + 3\sin^2 x = 2 + \sin^2 x$$

   is correct, an equality being a particular kind of statement.

   $$\sin^2 x + \cos^2 x \Rightarrow 1$$

   does not make sense: "$\sin^2 x + \cos^2 x$" and "1" are not statements.

   (c) The symbol " $\to$ " means *tends to*, as in "has limit". It cannot be substituted either to " $\Rightarrow$ " or " $=$ ".

2. Never write $0/0$, $a/0$, $1/\infty$, a/∞, ∞/∞, $0\times\infty$, $\ln(0)$, 0^0 or any other variant of this style. In particular, ∞ is not a number.

3. Distinguish between the expression of a function and its value at a particular point. If I found that $g(x) = 1 + x$ and I need its value at $x = 1$, I don't write

   $$g(x) = 1 + x = 2$$

   since "$g(x) = 1 + x$" is an identity (true for all x) but "$1 + x = 2$" certainly is not, but I write two separate statements:

   $$g(x) = 1 + x$$

   $$g(1) = 2\,.$$

   This is one more reason to name all functions, even if the data of the problem does not name them. If no name was assigned, it is still possible to distinguish the function and a particular value:

   $$(1 + x)|_{x=1} = 2\,.$$

   There are of course cases when it is appropriate to write "$1 + x = 2$": for example, as part of the sentence "Solve for x the equation $1 + x = 2$". But this is practically the only exception.

4.

(a) The lim symbol is never used by itself. So "$\lim_{t \to 0} = -1$" is not valid; you must specify limit of what.

(b) The lim symbol takes precedence over algebraic operators. So

$$\lim_{x \to 1} x^2 - x + 1$$

is read as "$(\lim_{x \to 1} x^2) - x + 1$", which is the function $-x + 2$, not at all the same as

$$\lim_{x \to 1} (x^2 - x + 1)$$

which is the number 1. Always use parentheses following the limit symbol when taking the limit of compound expressions. Even better, avoid using the limit symbol altogether; see following footnote.

5. Limit symbol, continued. Consider the function

$$f(x) = \frac{x^2 - 1}{x - 1}.$$

Find its limit as $x \to 1$:

$$\frac{x^2 - 1}{x - 1} = x + 1, \quad x \neq 1$$

$$\frac{x^2 - 1}{x - 1} \to 2 \text{ as } x \to 1 \qquad (1)$$

The limit, 2, is a fixed number and does not depend on x. In our use of notation, we distinguish fixed quantities from functions of some variable. Therefore the following is correct:

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} (1 + x)$$

$$= 2 \quad (2)$$

(fixed number = fixed number = fixed number), whereas in the following, both " = " signs are wrong:

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 1 + x$$

$$= 2 \quad (3)$$

(fixed number = function of x = fixed number).
(3) is a serious mistake. Style (2) is correct, but style (1) is better, since it explains in full why the function has limit 2 (almost everywhere equal to the function $x + 1$). For this reason, I

asked you to solve problems on limits without using the lim symbol. Use style (1).

6. As with any language, a solution in mathematics is a sequence of sentences, separated by periods. The sentences are often, but not always, themselves sequences of equalities (or inequalities). They may sometime contain words ("since the function is continuous, the hypothesis of such theorem holds…"). A "solution" which consists of a heap of disconnected expressions will be considered nonsensical, and may not get any credit.

7. Definition of derivative. If f is the function we want to differentiate,

$$D_x(h) = \frac{1}{h}(f(x+h) - f(x))$$

is called a difference quotient. It is the slope of a certain chord to the graph of f at the point $(x, f(x))$, not the slope of the tangent at that point. It depends on both x and h.
The derivative of f, which depends only on x, is the *limit* of $D_x(h)$ as $h \to 0$. So:

$$D_x(h) \to f'(x) \text{ as } h \to 0$$

is correct.

$$f'(x) = D_x(h)$$

is nonsense (expression depending on x only = expression depending on both x and h). This mistake is of the same kind as that discussed in [5] above.

8. Use of parentheses. Algebraic symbols $+$, $-$, $\cdot$, $\times$ etc… are not allowed to collide and must be separated by parentheses. To multiply x by $-y$, we write $x(-y)$. "$x \cdot -y$" or "$x \times -y$" is incorrect, even if you reduce the size of the minus sign, raise it and stick it very close to y. Also, $x - y$ and $x + (-y)$ are correct. "$x + -y$" is not, even if you reduce the size of the minus sign, raise it and stick it very close to y.

9. (Adventures with square root). To solve, say, $x^2 = 5$, some like to use as a step:

$$\sqrt{x^2} = \sqrt{5} \qquad (1)$$

Omit this step: in real numbers it is redundant, and in complex numbers it is defective. The correct steps are:

$$x^2 - 5 = 0$$

$$(x + \sqrt{5})(x - \sqrt{5}) = 0$$

yielding the solutions $x = -\sqrt{5}, x = \sqrt{5}$. This way, in complex numbers, $x^2 = -5$ gives:

$$x^2 + 5 = 0$$

$$(x - \sqrt{5}i)(x + \sqrt{5}i) = 0\ldots$$

As for (1), which I am asking you to omit, keep in mind that in real numbers, $\sqrt{x^2} \equiv |x|$, not x .