Some common corrections

1. Many are using the following symbols as if they meant the same thing:

   (a) The symbol " = " means *equals,* and nothing else. It connects expressions, sets, quantities, or any objects of the same nature, but not statements. It is not an all-purpose connector, nor does it open a paragraph.

   (b) The symbol " $\Rightarrow$ " means *implies.* It connects sentences or logical statements; it does not connect values or expressions. For instance:

   $$\sin^2 x + \cos^2 x = 1 \;\Rightarrow\; 2\cos^2 x + 3\sin^2 x = 2 + \sin^2 x$$

   is correct.

   $$\sin^2 x + \cos^2 x \Rightarrow 1$$

   does not make sense.

   (c) The symbol " $\to$ " means *tends to,* as in "has limit". It cannot be substituted either to " $\Rightarrow$ " or " = ".

2. We never use the integral symbol without the differential symbol. $\int f(x)$ is incorrect. If you are integrating with respect to x, you must write $\int f(x)\,dx$.

3. We never identify differential expressions with scalars:

   $$d(\cos\theta) = -\sin(\theta)d\theta$$

   is correct (on the right, scalar times differential = differential).

   $$d(\cos\theta) = -\sin(\theta)$$

   is not, and will steadily drain you of precious points on tests.

4. (Adventures with square root). Negative numbers are not the square root of their square. The "identity" $x \equiv \sqrt{x^2}$ is not correct. The correct one is

   $$\sqrt{x^2} \equiv |x|.$$

   In particular, $x = t^2 - 4$ means $x + 4 = t^2$, but does not mean $t = \sqrt{x+4}$, specially in a problem where it is specified that t takes negative values.

5. The comparison properties of the integral are the one thing you are required still to remember when you will have forgotten everything else about the integral. They are found at the end of §5.2 of Stewart. Upshots of these properties are:

   (a) If $f(x) \geq 0$ on an interval, then $\int f(x)\,dx$ on that interval cannot be negative.

   (b) If $f(x) \leq M$ on the interval (a, b), then $\int_a^b f(x)\,dx$ cannot exceed $M(b-a)$.

These facts hold not only for the exact integral, but for the numerical integration schemes as well. This means that if, for instance, the values of f at the integration node points do not exceed M, then the numerical approximation of $\int_a^b f\,dx$ cannot exceed $M(b-a)$.

6. Finding polar coordinates. The sytem of equations in (r, θ)

$$\begin{cases} r\cos\theta = a \\ r\sin\theta = b \end{cases} \qquad (1)$$

the single equation

$$\tan\theta = \frac{b}{a} \qquad (2)$$

and the equation

$$\theta = \arctan(b/a) \qquad (3)$$

(where, say, $r > 0$), are being used interchangeably (usually, transforming (1) into (3)). This is wrong, as each means something different:
(1) determines θ up to a multiple of 2π.
(2) determines θ up to a multiple of π.
(3) refers to a single angle in the interval $(-\pi/2, \pi/2)$.

In problem 1 of exam 3, where $a = \sqrt{3}, b = -1$, (1), along with the restriction $0 \le \theta < 2\pi$, gives $r = 2,\ \theta = 11\pi/6$. (2) is not enough to determine the angle, and (3) gives the wrong result ($\theta = -\pi/6$).

In practice, (3) is only to be used as a step, if you are asked for an approximate value, in conjunction with the use of table or calculator. The correct notation for (3) is:

$$\theta \sim \arctan(b/a)$$

which simply means: "$\arctan(b/a)$ is the arc in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ having the same tangent as θ".

7. Consider the sequence

$$a_n = \frac{n-1}{1+n}.$$

a_n is a function of n : $a_0 = -1,\ a_1 = 0,\ a_2 = 1/3$, etc... Find its limit: $\frac{n-1}{1+n} = \frac{1-1/n}{1+1/n} \to 1$ as $n \to \infty$. The limit, 1, is a fixed number and does not depend on n. In our use of notation, we carefully distinguish fixed quantities from functions of n. Therefore the following is correct:

$$\lim_{n\to\infty} \frac{n-1}{1+n} = \lim_{n\to\infty} \frac{1-1/n}{1+1/n}$$

$$= 1$$

(fixed number = fixed number = fixed number), whereas in the following, both " = " signs are wrong:

$$\lim_{n \to \infty} \frac{n-1}{1+n} = \frac{1-1/n}{1+1/n}$$

$$= 1$$

(fixed number = function of n = fixed number). This is a particular instance of the abuse of the $=$ sign referred to in note [1.a] above, since functions and fixed numbers are not objects of the same nature. This practice will be penalized.

8. Never write $0/0$, $a/0$, $1/\infty$, a/∞, ∞/∞, $0 \cdot \infty$ or any other variant of this style.