Some common corrections

1. Many are using the following symbols as if they meant the same thing:

   (a) The symbol " $=$ " means *equals*, and nothing else. It connects expressions, sets, quantities, or any objects of the same nature, but not statements. It is not an all-purpose connector, nor does it open a paragraph.

   (b) The symbol " $\Rightarrow$ " means *implies*. It connects sentences or logical statements; it does not connect values or expressions. For instance:

   $$\sin^2 x + \cos^2 x = 1 \;\Rightarrow\; 2\cos^2 x + 3\sin^2 x = 2 + \sin^2 x$$

   is correct.

   $$\sin^2 x + \cos^2 x \Rightarrow 1$$

   does not make sense: "$\sin^2 x + \cos^2 x$" and "1" are not statements.

   (c) The symbol " $\rightarrow$ " means *tends to*, as in "has limit". It cannot be substituted either to " $\Rightarrow$ " or " $=$ ".

2. We never use the integral symbol without the differential symbol. $\int f(x)$ is incorrect. If you are integrating with respect to x, you must write $\int f(x)\,dx$.

3. We never equate differential expressions with scalars:

   $$d(\cos\theta) = -\sin(\theta)d\theta$$

   is correct (on the right, scalar times differential = differential).

   $$d(\cos\theta) = -\sin(\theta)$$

   is not.

4. The comparison properties of the integral are the one thing you are required still to remember when you will have forgotten other details about the integral. They are found at the end of §5.2 of Stewart. Upshots of these properties are:

   (a) If $f(x) \geq 0$ on an interval, then $\int f(x)\,dx$ on that interval cannot be negative.

   (b) If $f(x) \leq M$ on the interval (a, b), then $\int_a^b f(x)\,dx$ cannot exceed $M(b-a)$.

   These facts hold not only for the exact integral, but for the numerical integration schemes as well. This means that if, for instance, the values of f at the integration node points do not exceed M, then the numerical approximation of $\int_a^b f\,dx$ cannot exceed $M(b-a)$.

5. Distinguish between the expression of a function and its value at a particular point. If I found that $g(x) = 1 + x$ and I will need its value at $x = 1$, I don't write

   $$g(x) = 1 + x = 2$$

since "$g(x) = 1 + x$" is an identity (true for all x) but "$1 + x = 2$" certainly is not, but I write two separate statements:

$$g(x) = 1 + x$$

$$g(1) = 2\,.$$

This is one more reason to name all functions, even if the data of the problem does not name them. If no name was assigned, it is still possible to distinguish the function and a particular value:

$$(1 + x)\big|_{x=1} = 2\,.$$

There are of course cases when it is appropriate to write "$1 + x = 2$": for example, as part of the sentence "Solve for x the equation $1 + x = 2$". But this is practically the only exception.

6. Use of parentheses. Algebraic symbols $+$, $-$, $\cdot$, $\times$ etc… are not allowed to collide and must be separated by parentheses. To multiply x by $-y$, we write $x(-y)$. "$x \cdot -y$" or "$x \times -y$" is incorrect, even if you reduce the size of the minus sign, raise it and stick it very close to y. Also, $x - y$ and $x + (-y)$ are correct. "$x + -y$" is not, even if you reduce the size of the minus sign, raise it and stick it very close to y.

7. Indefinite integrals are (classes of) functions: $\int (1 + \tan^2\theta)\,d\theta = \tan\theta + C$. Definite integrals are numbers (fixed values): $\int_0^{\pi/4} (1 + \tan^2\theta)\,d\theta = 1$. This difference carries over to the notation: the first might be denoted $I(\theta)$, the second, I.

8. In words problems, never include physical units in the intermediate steps, and never inside integrals. Physical units appear twice, like the gun on the mantelpiece in the film noir: at the beginning, where you define symbols ("let V be the volume in cubic centimeters") and at the end ("the total work is 2.5 J").

9. (Adventures with square root). To solve, say, $x^2 = 5$, some like to use as a step:

$$\sqrt{x^2} = \sqrt{5} \qquad (1)$$

Omit this step: in real numbers it is redundant, and in complex numbers it is defective. The correct step is:

$$x^2 - 5 = 0$$

$$(x + \sqrt{5})(x - \sqrt{5}) = 0$$

yielding the solutions $x = -\sqrt{5}$, $x = \sqrt{5}$. This way, in complex numbers, $x^2 = -5$ gives:

$$x^2 + 5 = 0$$

$$(x - \sqrt{5}i)(x + \sqrt{5}i) = 0\ldots$$

As for (1), which I am asking you to omit, keep in mind that in real numbers, $\sqrt{x^2} \equiv |x|$, not x.

10. Trigonometric substitutions. If you encounter in an integral the expression $x^2 - 1$ and you decide to replace x by $\sec\theta$, you must also specify the interval of θ, as we have done in class. "$x = \sec\theta$" by itself only lets $\sqrt{\tan^2\theta} = |\tan\theta|$ (see note [9]). If you want to use $\sqrt{\tan^2\theta} = \tan\theta$, you must write:

$$x = \sec\theta, \ \ \theta \in [0, \pi/2) \cup [\pi, 3\pi/2).$$

The same holds for any other trigonometric substitution: $x = \sin\theta, \ -\pi/2 \le \theta \le \pi/2$, etc...

11. Using the "inverse trigonometric functions". The equations

$$r = \sin\theta \qquad (1)$$

$$\theta = \sin^{-1}(r) \qquad (2)$$

are not equivalent. If, say, the angle θ and t, time in years, are linked by $t = \theta$, and (1) gives the position of a particle at time t, then (1) tells us that 2000 years ago, the particle was at the point of coordinates (0.3418, 0.8650) on a certain circle. (2), on the other hand, does not let t take values outside the interval $[-\pi/2, \pi/2]$.

The functions $\sin^{-1}, \cos^{-1}, \tan^{-1}$ are only useful as a step in certain numerical problems. They play no role in the polar equations of curves. On tests, avoid using them altogether, and leave them in your bag, along with cell phone and calculator.

12. Never write $0/0$, $a/0$, $1/\infty$, a/∞, ∞/∞, $0 \times \infty$, $\ln(0)$, 0^0 or any other variant of this style.

13. Three dots make a difference. The expressions

$$a_1 + a_2 + a_3 + a_4 \qquad (1)$$

$$a_1 + a_2 + \ldots + a_n \qquad (2)$$

$$a_1 + a_2 + \ldots + a_n + \ldots \qquad (3)$$

each means something different. (1) denotes a sum of four terms. (2) is a sum of n terms, hence depends on the integer n. It is the n-th partial sum of the series $\sum_{k \ge 1} a_k$. (3) on the other hand denotes the series itself (or its sum). In particular, do not confuse (2) and (3).