MATE 3032-011 First examination September 6, 2017

Name: score:

Write only on one side of each sheet. No calculator, as exact values are required where asked for. In those problems where you have to find a volume, sketch the region, the solid, and a typical disc, annulus (washer) or cylindrical shell.

1.

(a) Find the average value of $f(x) = \sqrt{x^2 - 1}/x,\ 1 \le x \le 7$.

(b) Find $\int \sin^3(\theta)\cos^4(\theta)\, d\theta$.

2.

(a) Find the volume generated by rotating the region bounded by the curves

$$y = \sqrt[3]{x},\ y = 0,\ x = 1$$

about the y-axis.

(b) Set up an integral for the volume of the solid obtained by rotating the region bounded by

$$y = \tan x,\ y = 0,\ x = \pi/4$$

about the line $x = \pi/2$, using cylindrical shells. Do not compute the integral.

3. Find the volume of the solid obtained by rotating the region bounded by

$$y = \ln x,\ y = 1,\ y = 2,\ x = 0$$

about the y-axis.

4. A heavy rope, 50 ft long, weighs 0.5 lb/ ft and hangs over the edge of a building 120 ft high. How much work is required to pull the rope to the top of the building?

5. Evaluate $\int_1^2 x^3 \ln x\, dx$.

6. Suppose you push a book across a 6-meter-long table by exerting a force $F(x)$ at each point from $x = 0$ to $x = 6$. What does $\int_0^6 F(x)\, dx$ represent? if $F(x)$ is measured in newtons, what are the units for the integral?