Name: score:

Write only on one side of each sheet. No calculator, as exact values are required where asked for. In those problems where you have to find a volume, sketch the region, the solid, and a typical disc, annulus (washer) or cylindrical shell.

1. An aquarium 2 m long, 1 m wide, and 1 m deep is full of water. Find the work needed to pump half of the water out of the aquarium. The density of water is 1000 kg/m^3.

2.

(a) Evaluate $\int (\tan^2 t + \tan^4 t)\, dt$.

(b) Evaluate $\int_0^{\pi/2} \dfrac{\cos t}{\sqrt{1 + \sin^2 t}}\, dt$.

3.

(a) Evaluate $\int \cot^5\theta \sin^4\theta\, d\theta$.

(b) Evaluate $\int t \sin^2 t\, dt$.

4. Set up the integral which gives the volume of the solid obtained by rotating the region bounded by

$$y = \frac{x^2}{4},\ y = 5 - x^2$$

about the x-axis. Do not compute the integral.

5. Find $\int_{\pi/4}^{\pi/3} \dfrac{\sqrt{\tan\theta}}{\sin(2\theta)}\, d\theta$. (Hint: let $t = \tan\theta$.)

6. Each integral represents the volume of a solid. Describe the solid:

(a) $\int_0^{\pi} \pi(2 - \sin x)^2\, dx$ (b) $\int_0^{\pi/2} 2\pi x \cos x\, dx$.