

Name:

score:

Write only your name on the cover sheet. Write on one side only of each blank sheet. No calculator, as exact values are required where asked for.

1.

(a) $2\pi \int_0^2 \frac{y}{1+y^2} dy$ represents the volume of a solid. Describe and sketch the solid.

(b) Find $\int_{\pi/4}^{\pi/3} \frac{\sqrt{\tan\theta}}{\sin(2\theta)} d\theta$. (Hint: let $t = \tan\theta$.)

2. Consider the curve $x = \sin t$, $y = \csc t$, $0 < t < \pi/2$.

(a) Eliminate the parameter t so as to get a cartesian equation for the curve.

(b) Sketch the curve, and indicate in which sense it is traced as t increases.

3. Consider the curve

$$x = 3t - t^3, \quad y = 3t^2.$$

Find the length of the loop.

4. Find the volume of the solid obtained by rotating the region bounded by $y = \sin x$, $y = 0$, $x = \pi/2$, $x = \pi$ about the x -axis.

5. Give both cartesian and polar equations for the circle of centre $(2, -3)$ and radius 4.

6.

(a) If the infinite curve $y = e^{-x}$, $y \geq 0$ is rotated about the x -axis, find the area of the corresponding surface.

(b) Use the comparison theorem to determine whether

$$\int_0^{\pi} \frac{\sin^2 x}{\sqrt{x}} dx$$

is convergent or divergent.