

Name:

score:

Write only your name on the cover sheet. Write on one side only of each blank sheet. No calculator, as exact values are required where asked for.

1. Find the volume of the solid obtained by rotating about the  $y$ -axis the region under the graph of  $y = x\sqrt{1-x^2}$ ,  $0 \leq x \leq 1$ . (Sketch the region).
2.
  - (a) Find a cartesian equation of the curve given in parametric form by  $x = 1 + e^{2t}$ ,  $y = e^t$ ,  $-\infty < t < \infty$ . Sketch the curve (and be careful not to put too much).
  - (b) Give three different sets of parametric equations for the curve  $y = \sqrt{x}$ .
3. Consider the surface obtained by rotating the curve  $y = 1/x$ ,  $1 \leq x < \infty$  about the  $x$ -axis (*Gabriel's horn*). Sketch it. Show that its (surface) area is infinite.
4.
  - (a) Find cartesian equations for the curves given in polar coordinates by  $r = 4\sin \theta$ ,  $r = 2$ .
  - (b) Find the area of the region inside  $r = 4\sin \theta$  and outside  $r = 2$ .
5.
  - (a) Suppose that you know that the sequence  $(a_n)$  is decreasing, and that  $3 \leq a_n \leq 8$  for each  $n$ . Must the sequence have a limit? If yes, what can you say about the value of the limit?
  - (b) Find the limit of the sequence  $(n^2 e^{-n})$ .
6.
  - (a) Find the sum of  $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$ .
  - (b) For which values of  $x$  does  $\sum_{n \geq 1} \ln(x)^n$  converge?