

Name:

score:

Write only your name on the cover sheet. Write on one side only of each blank sheet. No calculator, as exact values are required where asked for.

1. Find a polar equation for the curve represented by the cartesian equation:

(i) $x + y = 2$

(ii) $x^2 + y^2 = 2$

2.

- (a) If the infinite curve $y = e^{-x}$, $x \geq 0$ is rotated about the x -axis, find the area of the corresponding surface.

- (b) Use the comparison theorem to determine whether $\int_0^{\pi} \frac{\cos^2 x}{\sqrt{x}} dx$ is convergent or divergent.

3. For which values of t is the curve $x = t + \ln t$, $y = t - \ln t$ convex (concave upward)?

4. Find whether the statement is true or false. If true, explain why. If false, explain why, or give an example disproving the statement.

- (a) The ratio test can be used to determine convergence of $\sum 1/n^3$.

- (b) The ratio test can be used to determine convergence of $\sum 1/n!$.

- (c) If $\sum a_n$ diverges, so does $\sum |a_n|$.

5.

- (a) Suppose $\sum a_n$ and $\sum b_n$ are positive series, and $\sum b_n$ is known to be divergent.

- (i) If $a_n < b_n$ for all n , what can you say about $\sum a_n$? Why?

- (ii) If $a_n > b_n$ for all n , what can you say about $\sum a_n$? Why?

- (b) Determine whether $\sum_{n=0}^{\infty} \frac{\sqrt{1+n}}{2+n}$ converges or diverges.

6. Test for convergence or divergence:

(i) $\sum_{n \geq 1} (-1)^n \frac{\ln(n)}{\sqrt{n}}$

(ii) $\sum_{k \geq 0} \frac{4^k}{3^k + 5^k}$

A formula

$$\int \sec^3 \theta d\theta = \frac{1}{2}(\sec \theta \tan \theta + \ln|\sec \theta + \tan \theta|)$$