Some common corrections

1. Many are using the following symbols as if they meant the same thing:

   (a) The symbol " $=$ " means *equals*, and nothing else. It connects expressions, sets, quantities, or any objects of the same nature, but not statements. It is not an all-purpose connector, nor does it open a paragraph.

   (b) The symbol " $\Rightarrow$ " means *implies*. It connects sentences or logical statements; it does not connect values or expressions. For instance:

   $$\sin^2 x + \cos^2 x = 1 \;\Rightarrow\; 2\cos^2 x + 3\sin^2 x = 2 + \sin^2 x$$

   is correct.

   $$\sin^2 x + \cos^2 x \Rightarrow 1$$

   does not make sense: "$\sin^2 x + \cos^2 x$" and "1" are not statements.

   (c) The symbol " $\to$ " means *tends to*, as in "has limit". It cannot be substituted either to " $\Rightarrow$ " or " $=$ ".

2. We never use the integral symbol without the differential symbol. $\int f(x)$ is incorrect. If you are integrating with respect to x, you must write $\int f(x)\,dx$.

3. Use of parentheses. Algebraic symbols $+$, $-$, $\cdot$, $\times$ etc… are not allowed to collide and must be separated by parentheses. To multiply x by $-y$, we write $x(-y)$. "$x \cdot -y$" or "$x \times -y$" is incorrect, even if you reduce the size of the minus sign, raise it and stick it very close to y . Also, $x - y$ and $x + (-y)$ are correct. "$x + -y$" is not, even if you reduce the size of the minus sign, raise it and stick it very close to y .

4. We never equate differential expressions with scalars:

   $$d(\cos\theta) = -\sin(\theta)d\theta$$

   is correct (on the right, scalar times differential = differential).

   $$d(\cos\theta) = -\sin(\theta)$$

   is not.

5. The comparison properties of the integral are the one thing you are required still to remember when you will have forgotten other details about the integral. They are found at the end of §5.2 of Stewart. Upshots of these properties are:

   (a) If $f(x) \geq 0$ on an interval, then $\int f(x)\,dx$ on that interval cannot be negative.

   (b) If $f(x) \leq M$ on the interval (a, b), then $\int_a^b f(x)\,dx$ cannot exceed $M(b - a)$.

   These facts hold not only for the exact integral, but for the numerical integration schemes as well. This means that if, for instance, the values of f at the integration node points do not exceed M, then the numerical approximation of $\int_a^b f\,dx$ cannot exceed $M(b-a)$.

6. In problems involving finding the work, all of you start with

$$W = F \cdot \text{distance}.$$

This is incorrect, as it only makes sense if F is constant over the distance travelled. In all the examples we gave, this was the case only once: lift a book from the floor, and assume that the pull of gravity does not decrease with height. The correct relation is one of

$$dW = F \cdot d(\text{distance})$$

or

$$dW = \text{distance} \cdot dF$$

and it is part of your solution to specify which.

7. Indefinite integrals are (classes of) functions: $\int (1+\tan^2\theta)\,d\theta = \tan\theta + C$. Definite integrals are numbers (fixed values): $\int_0^{\pi/4} (1 + \tan^2\theta)\,d\theta = 1$. This difference carries over to the notation: the first might be denoted $I(\theta)$, the second, I.

8. As with any language, a solution in mathematics is a sequence of sentences, separated by periods. The sentences are often, but not always, themselves sequences of equalities (or inequalities). They may sometime contain words ("since the function is continuous, the hypothesis of such theorem holds…"). A "solution" which consists of a heap of disconnected expressions will be considered nonsensical, and may not get any credit.

9. Trigonometric substitutions. If you encounter in an integral the expression $x^2 - 1$ and you decide to replace x by $\sec\theta$, you must also specify the interval of θ, as we have done in class. "$x = \sec\theta$" by itself only lets $\sqrt{\tan^2\theta} = |\tan\theta|$. If you want to use $\sqrt{\tan^2\theta} = \tan\theta$, you must write:

$$x = \sec\theta,\ \ \theta \in [0, \pi/2) \cup [\pi, 3\pi/2),$$

since on these intervals, $\tan\theta \ge 0$. (And you must explain all these steps). The same holds for any other trigonometric substitution: $x = \sin\theta,\ -\pi/2 \le \theta \le \pi/2$, etc…

10. This is not a correction, but a useful topic: how to derive the formulæ for the cosine or sine of sums of angles from Euler's formula. The elements are:

    (1) Euler's formula:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

    (2) Multiplication of complex numbers: same as that of real numbers, using the fact that $i^2 = -1$:

$$(u + iv)(x + iy) = ux + iuy + ivx + i^2vy$$

$$= ux - vy + i(uy + vx)$$

so the real part is $ux - vy$, the imaginary part is $uy + vx$.

(3) Multiplicative property of the exponential function: the formula

$$e^{x+y} = e^x e^y$$

holds also for complex x, y, in particular for purely imaginary numbers:

$$e^{ia+ib} = e^{ia} e^{ib} .$$

(4) Complex numbers are equal only when their real part and their imaginary part are the same: so

$$x + iy = u + iv$$

(x, y, u, v real) implies $x = u, y = v$.

To obtain the formula for $\cos(a + b)$, write e^{ia+ib} in two different ways:

$$e^{i(a+b)} = e^{ia}e^{ib}$$

or:

$$\cos(a + b) + i \sin(a + b) = (\cos a + i \sin a)(\cos b + i \sin b) \qquad (1)$$

The term on the left has real part $\cos(a + b)$, the term on the right, $\cos a \cos b - \sin a \sin b$. Identifying them gives:

$$\cos(a + b) = \cos a \cos b - \sin a \sin b .$$

Note that identifying the imaginary parts of both sides of (1) also gives the formula for $\sin(a + b)$.
Once you have this formula, the one for $\cos(a - b)$ results by replacing b by $-b$. Combining the two by adding them, you can express $\cos a \cos b$ in terms of $\cos(a + b)$ and $\cos(a - b)$. A similar manipulation allows expressing $\sin a \cos b$ in terms of $\sin(a + b)$, $\sin(a - b)$. See Stewart p. 484.

11. Never write $0/0$, $a/0$, $1/\infty$, a/∞, ∞/∞, $0 \times \infty$, $\ln(0)$, 0^0 or any other variant of this style. In particular, ∞ is not a number.