Exam 1-060

1. Let $u = \sqrt{x-1}$.

2.

   (a) We used the identity to write two equations: one for $\sin(a+b)$, the second for $\sin(a-b)$, then we combined the two.

   (b) Let $t = \theta/2$.

3. First step is let $x = e^t$. Second step is a trigonometric substitution, where the solution mirrors what you find in CC10. Read again CC10, as I will ask most of you to redo problems 31-32 of homework, including figure and including such words as "then" and "since".

4. Do not forget the figures.

Exam 1-071

1. See CC9.

2.b) Complete the square, then use a trigonometric substitution, mirroring the steps shown in CC10.

3. The integrand is a not rational function of x, but of $\sqrt{x}$.

Exam 1-080

1.b) The mean-value theorem and the mean-value theorem for integrals are different theorems. Both have a statement which must include the equivalent of the words "for some" or "there exists".

2.b) To show convergence using comparison, you may bound the integrand from above by a function of the form $\frac{C}{x\sqrt{x}}$, where C is a positive constant to be determined. Does $C = 1$ work?

4.b) First complete the square, then use a trigonometric substitution, mirroring the steps shown in CC10.

Exam 2

1.

(a), (b): the figure in CC10 shows those arcs where $\tan\theta \geq 0$, which is also where $\sqrt{\tan^2\theta} = |\tan\theta| = \tan\theta$. The loops indicate that $\tan\theta$ is not defined at $\theta = \pm\pi/2$. On the other hand, $\sin\theta$ is defined everywhere, including the semi-circle where $\sin\theta \leq 0$.

Part (c) was discussed in class: the equations

$$\tan\theta = y/x \qquad (1)$$

$$\theta = \arctan(y/x) \qquad (2)$$

are not equivalent. The first has infinitely many solutions θ, whereas the second only allows $-\pi/2 < \theta < \pi/2$. Furthermore, neither is a valid substitute for the exact system

$$r\cos\theta = x,\ \ r\sin\theta = y \qquad (3)$$

Equation (2) is only useful when using a table or calculator, and, in order to find the correct solutions, has to be used in conjunction with (3). When looking for exact values, the only useful form is (3), which will give you infinite families of solutions (r, θ). Keep also in mind that r is allowed negative values.

(d)–(g) Among the tools we have to determine convergence of series, one is the principle of comparison between positive series, another, the theorem which states that an absolutely convergent series is convergent.

2.a) The equations

$$r^2 = 3r \qquad (1)$$

$$r = 3 \qquad (2)$$

are not equivalent. (2) represents a circle centred at the origin; (1) represents a larger set: the same circle, plus the origin itself, since (1) amounts to

$$r(r-3) = 0\,.$$

Going from (1) to (2) by “simplifying by r”, as folk wisdom (or superstition) tricks you into, is a serious mistake, since you are deleting the valid solution $r = 0$. In problem 57 of homework, the same mistake was made, replacing

$$\sin\theta = 2\sin\theta\cos\theta$$

by

$$2\cos\theta = 1\,.$$

4.

(a) A common mistake is the misuse of parentheses. Keep in mind that $(2n)! \neq 2n! = 2(n!)$.

Here, if $a_n = \dfrac{(2n)!}{(n!)^2}$, then $a_{n+1} = \dfrac{(2n+1)(2n+2)}{(n+1)^2}\, a_n$.

(b) To show that this series is an alternating series for $n \geq$ some p, you must, as a step, study the variation of $f(x) = x^2 e^{-x}$. For this step, I have usually shown the graph of a function which has the same sign as $f'(x)$. Do the same.

(c) A series will converge if it has the alternating series property, but the property is not necessary for convergence. So, in addition to finding the values of p for which the series is alternating for k large enough (here as it happens, $k \geq 1$), you must also show that for other values of p, the series will diverge.

5. Let $\varepsilon = 0.5 \times 10^{-2}$. The error bound for alternating series tells us that $\left|\Sigma c_n - \sigma_5\right| \leq \left|c_6\right| \leq \varepsilon$. The lower estimate from the integral test tells us that $\left|\Sigma a_n - s_8\right| \geq \int_9^\infty 1/t^3 \, dt > \varepsilon$.