MATE 3032 assignment 12: sections 11.10, 11.3, 11.5

104. Exercise 2 p.771.

105. Using the definition of Taylor series, find the first four nonzero terms of the series of $f(x)$ centred at a:

(a) $f(x) = \cos^2 x,\ a = \pi$

(b) $f(x) = \dfrac{1}{1+x},\ a = 1$.

106. Find the Taylor series of $f(x)$ centred at a:

(a) $f(x) = 1/x,\ a = -2$

(b) $f(x) = e^{-3x},\ a = 0$.

107. Using the binomial series, find the Maclaurin series of $f(x)$ and state the radius of convergence:

(a) $f(x) = \dfrac{x^2}{\sqrt{4+x}}$

(b) $f(x) = \sqrt[3]{8-x}$.

108. Use multiplication or division of series to find the first three nonzero terms of the Maclaurin series of:

(a) $e^x \ln(1-x)$

(b) $x/\sin x$.

109-110. Exercises 66, 78 p.772.

The remaining problems are about error bound. Recall that we have encountered two ways of bounding the error in the sum of a series: for positive series where the integral test applies, a remainder of the series is bounded by a tail of the integral. For alternating series, the size (absolute value) of the remainder is bounded by the modulus of the next term. Another kind of error is that between the n-th Taylor polynomial and the function it approximates; a bound is given by the Taylor inequality. When x is fixed, the Taylor inequality is often simple enough to use.

111. Use series to approximate $\int_0^{0.5} x^2 e^{-x^2}\,dx$ to three correct decimal digits (this means that the error is in absolute value $\le 1/2 \times 0.001$).

112. Consider the series $\sum a_n$, where $a_n = \dfrac{\sin(40n)}{(1 + 100n)\sqrt{\ln n}}$. Let $s_n = \sum_{k=2}^{n} a_n$.

(a) Given that, to the eighth decimal digit, $s_{100} = -0.00426740$, compute $s_{101}, s_{102}, s_{103}, s_{104}$.

(b) Do you conclude that at the limit, the third decimal digit of $\sum_{n=2}^{\infty} a_n$ will stabilise to 4?

113. Consider the series $\sum_{n \geq 1} a_n$, where $a_n = 1/n^4$.

(a) Find a bound on the error committed by approximating the sum of the series by the n-th partial sum $s_n = \sum_{k=1}^{n} a_k$. The error will depend on n.

(b) What should be the value of n so that the error does not exceed 10^{-5}? Do not compute the corresponding s_n.

114-115. Choose two exercises from 13–22 on p.780 of the text.

116-117. Exercises 25, 26 p.781.