MATE 3032 assignment 7: sections 10.3, 10.4, 11.1

52. Find the points on the curve $r = 1 + 2\sin\theta$ where the tangent is horizontal or vertical.

53. Find the slope of the tangent line to the polar curve $r = 2\sin\theta$ at the point where $\theta = \pi/4$.

54. Sketch the graph of r as a function of θ in cartesian coördinates, then sketch the curve given by the polar equation:

    (a) $r = 2(1 - \cos\theta)$

    (b) $r^2\theta = -1$.

Provide a figure for each of problems 55–58.

55. Find the area enclosed by one loop of the strophoid

$$r = 2\cos\theta - \sec\theta.$$

56. Find the area of the region which lies inside both curves:

    (a) $r = \cos(2\theta),\ r = \sin(2\theta)$

    (b) $r = 2\cos\theta, r = \cos\theta$ and bounded by the rays $\theta = 0, \theta = \pi/4$.

57. Find all points of intersection of the two curves $r = \sin\theta,\ r = \sin(2\theta)$.

58. Exercise 44 p. 673.

59. Determine whether the sequence converges or diverges. If it converges, find the limit.

    (a) $a_n = \dfrac{(-1)^n n}{n + 2\sqrt{n}},\ n \geq 1$

    (b) $(a_n) = (0, 1, 0, 0, 1, 0, 0, 0, 1, \ldots)$. Hint: can a strip of diameter smaller than one contain a remainder of the sequence?

    (c) $a_n = \cos n$.

60. Prove that if $a_n \to 0$ and (b_n) is bounded, then $a_n b_n \to 0$. Hint: use the squeeze theorem. (b_n) is bounded just in case $|b_n| \leq M$ for some M.

61. Graph the sequence

$$a_n = (-1)^n \frac{n}{2n+1}$$

to decide whether it converges or diverges. If it converges, guess the limit from the graph, then prove your guess:

i) using directly the definition (this is a good exercise to test your understanding of the definition)
ii) using the limit rules.

62. Exercise 83 p.706.