

MATE 3063 Practice exercises

Functions

All exercises are from section 14.1 (p. 865 and following) of the text.

1–4: exercises 6,7,8,10. When determining the range, give a complete proof. To show that two sets are equal, one must establish two set inclusions (one going each way). We have seen two methods to find the range: one is to find a subset (usually, some parametrised curve) with image filling the range; the other, when applicable, is to use the level sets. The two can be neatly combined: knowing the level sets tells you which curve to choose.

5–8: exercises 11,12,14,15

9–12: exercises 22,24,25,29

13–17: exercises 39,41,42,43,45.

Limits

Referring to the two-person game which we use for the definition of limit, show that the limit of the function in each case is the value given. It is enough, in each case, to establish the value of ρ (radius of player two's square) corresponding to:

i) Player one choosing $r = 1$

ii) Player one choosing arbitrary $r > 0$.

18. $\lim_{(x,y) \rightarrow (2,2)} 2x + y = 6$

19. $\lim_{(x,y) \rightarrow (2,-1)} x + 3y = -1$

20. $\lim_{(x,y) \rightarrow (\pi/4, \pi/4)} \cos x + \sin y = \sqrt{2}$.

21. $\lim_{(x,y) \rightarrow (0, \pi/2)} \cos x - 2\sin y = -1$.

Using the algebra of limits, continuity of known types of functions, and, if needed, the squeeze theorem, (but not directly the definition, as you did for exercises 18–20 above), find the limit:

22. $\lim_{(x,y) \rightarrow (1,2)} (5x^3 - x^2y^2)$

23. $\lim_{(x,y) \rightarrow (-1,1)} e^{-xy} \cos(x+y)$

24. (Corrected) $\lim_{(x,y) \rightarrow (0,0)} (xy \sin y)/(3x^2 + y^2)$

25. $\lim_{(x,y) \rightarrow (0,0)} (x^2 \sin^2 y)/(x^2 + 2y^2)$

For 24, use the inequality $|xy/(x^2 + y^2)| \leq \frac{1}{2}$.

Partial derivative

Section 14.3 (pp. 888–891) of Stewart.

26–27: exercises 10, 11.

28–34: exercises 20,21,24,28,31,37,38. For the last two, refer to the derivative as $\partial u / \partial x_k$, $1 \leq k \leq n$.

35–38: 39,42,43,44. For the last two, use the definition of partial derivative as limit, but to find the limit, use the algebra of limits (including continuity), but not the definition of limit.

39–48: 46,48,49,50,53,59,60,61,62,63.

49–51: 81,86,87.

Linear approximation

Section 14.4 of the text.

52–59: 2,4,6,11,13,15,19,21.

Chain rule

Section 14.5.

60–70: 2,3,8,10,15,16,22,23,43,46,50. Hint for the last two: multiply the equation by e^{2s} .

Directional derivative

Section 14.6.

71–85: 2,6,7,10,15,18,22,24,28,34,36,40,52,55,59 (hint: the tangent line to that curve belongs to two tangent planes).

86: Show that there is no point (x_0, y_0) , and no direction u , where the directional derivative $D_u f(x_0, y_0) = \sqrt{50}$, where $f(x, y) = \sin(\pi x) - \cos(\pi y) + \sin(x + y)$.

Additional exercises

87–88: repeat practice exercises 1 and 3, this time showing the complete steps to find the range. See the file “Pointers and corrections” for a complete example.

Extrema

Section 14.7

89–98: 2,3,6,13,17,19,42,43,53,54.

Integral over rectangles

§ 15.1. 99–105: 5, 9, 11, 13, 14, 17, 18.

106: Compute $\iint_D f(x, y) dA$, where D is the disc of radius 2 and centre the origin, and $f(x, y) = 2$ if $x + 2y < 0$, $f(x, y) = -1$ if $x + 2y > 0$. What kind of functions does f belong to? Draw an appropriate picture. Note that the values of f on the line $x + 2y = 0$ have no effect on the value of the integral, fact which we have not proven, but which you may accept for now.

Iterated integral

§ 15.2. 107–118: 2, 6, 7 (hint: do not expand), 17, 18, 23, 24, 25, 31, 35, 36, 38.

Integral over general regions

§ 15.3. 119–135: 1, 7, 10, 21, 23, 32, 33, 34, 41, 45, 52, 53, 54, 55, 56, 58, 61.

Polar coördinates

§ 15.4. 136–148: 4, 6, 7, 14, 18, 19, 21, 22, 28, 29, 35, 36, 37.

Triple integral

§ 15.6. 149–159: exercises 3, 8, 12, 13, 17, 22, 26, 28, 32, 33, 36.

Vector fields

§ 16.1. 160–177: exercises 2, 6, 8, 11–18, 22, 23, 29–32, 33.

178: the field of attraction to a unit point mass at the origin is given by

$$F = -\frac{mG}{|x|^3}x, \quad x \in R^3.$$

Find its domain and range.

Surface integral

§ 16.7. 179–186: 4, 9, 10, 11, 16, 19, 23, 29.

Divergence theorem

§ 16.9. 187–192: 3, 4, 9, 11, 12, 17.