

Name:

score:

1. (20) Given  $f(x, y) = \sqrt{xy}$

(a) Find and shade the domain.

(b) Explain why  $f$  is differentiable at  $(2, 2)$ , and find its linearisation at that point.

2. (20) Identify and sketch the graph of the surface  $x^2 = z^2 + 4y^2$ .

3. (30)

(a) Explain why the function  $g(x, y) = \frac{xy^2}{x^2 + y^4}$  has no limit as  $(x, y) \rightarrow (0, 0)$ .

(b) What can you say about the limit as  $(x, y) \rightarrow (0, 0)$  of  $\frac{xy^2}{x^4 + y^2}$ ?

4. (30)

(a) If  $z = xy + xe^{y/x}$ , show that

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = xy + z.$$

Choose one of (b) or (c):

- (b) An ellipsoid is obtained by rotating the ellipse  $x^2 + 4y^2 = 16$  about the  $x$ -axis. Find the equation of the ellipsoid.
- (c) A surface consists of all points at equal distance from the plane  $x = 1$  and the  $z$ -axis. Find its equation, identify it and sketch it.