

Name:

score:

Write only your name on the cover sheet. Write on one side only of each blank sheet.

1. Use traces to sketch and identify the surface $x = y^2 - z^2$. What symmetries does it have?
2. You may solve (b) using the result of (a).
 - (a) Show that the equation $x^2 + xy + y^2 = 0$ has only one solution (x, y) in the plane (complete a square).
 - (b) Determine the set of points where the function

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + xy + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

is continuous. Justify.

3.
 - (a) Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$: $z = f(x) + g(y)$.
 - (b) Find all the second partial derivatives of $w = \sqrt{1 + uv^2}$.
 - (c) You are told that there is a function f with partial derivatives $f_x(x, y) = x + 4y$ and $f_y(x, y) = 3x - y$. Should you believe it?
4. Let $f(x, y) = 1 - xy\cos(\pi xy)$.
 - (a) Explain why f is differentiable.
 - (b) Find the linear approximation of f at $(1, 1)$ and use it to approximate $f(1.03, 0.98)$.
5. Find the centre and the radius of the sphere of equation

$$x^2 + y^2 + z^2 - 4x + 2y - 6z + 9 = 0.$$

6. Find and shade the domain of the function

$$f(x, y) = \sqrt{4 - x^2 - y^2} + \sqrt{1 - x^2}.$$