MATE 3063 assignment 4: sections 14.4, 14.1

Problems 1–7 are from §14.4.

1. Find an equation of the tangent plane to the given surface at the given point:

   (a) $z = \dfrac{y+1}{x^2}$, $(2, -1, 0)$.

   (b) $z = \dfrac{1-x}{1+y}$, $(0, 0, 1)$.

   (c) $z = x \ln(x - 2y)$, $(3, 1, 0)$.

2. Given a function and a point, find the linearisation $L(x, y)$ of the function at that point:

   (a) $f(x,y) = \sqrt{xy}$, $(2, 2)$.

   (b) $f(x,y) = x \cos(x + y)$, $(1, -1)$.

   (c) $f(x,y) = y + \sin(x/y)$, $(0, 2)$. Use the linearisation in order to find an approximate value of $0.98 + \sin(0.1/0.98)$.

3. Problem 22 of text.

4. (a,b) Choose two of problems 25–30 of text.

5. (a,b) Problems 31, 35.

6. Exercise 42 of text: each curve, being a differentiable vector function of t, has a *tangent vector* at P (see p.856). The tangent plane at P contains both tangent vectors; so letting the vectors be v, w, the tangent plane has normal vector perpendicular to both v and w. To find v and w, you must first find the values of the parameters corresponding to the given point. There may be a different value of the parameter on each curve.

7.

   (a) Around the point $(1, 0)$, is $f(x, y) = x^2(y + 1)$ more sensitive to changes in x or to changes in y? Justify.

   (b) What ratio of dx to dy will make df equal zero at $(1, 0)$?

8. Draw a contour map, marking the level values. Use colours. Show at least 4-5 curves, displaying different possible cases. Also, show the intermediate step (y function of x or x function of y).

   (a) $f(x, y) = xy^2$.

   (b) $g(x, y) = y(x + 1) - 3$.

9. Using the test-and-shade method, find the domain:

(a) $f(x, y) = \sqrt{x - y} + \ln(x + y - 2)$.

(b) $g(x, y) = \sqrt{\ln(x + y - 2)}$.