

### MATE 4000 assignment 3

11. In general in a topological space, if  $A, B$  are arbitrary subsets, one of the sets  $(A \cup B)^o$  and  $\overset{o}{A} \cup \overset{o}{B}$  is always contained in the other. Show which. Give also an example where the sets are not equal.
12. Let  $\mathcal{C}$  be a collection of subsets of the set  $E$ . Suppose that  $\emptyset$  and  $E$  are in  $\mathcal{C}$ , and that finite unions and arbitrary intersections of elements of  $\mathcal{C}$  are in  $\mathcal{C}$ . Show that the collection

$$\mathcal{T} = \{E \setminus C : C \in \mathcal{C}\}$$

is a topology on  $E$ .

13. If  $X$  is a topological space and  $\mathcal{B}$  a filter base in  $X$ , show that the set of supersets of  $\mathcal{B}$  (sets containing an element of  $\mathcal{B}$ ) is a filter.
14. Problem 1, chapter II p. 129 from Dixmier.