Tips and common corrections

[1] Some comments about notation.

(a) The symbol " $=$ " means *equals*, and nothing else. Points are lost on tests and quizzes every semester for using it inappropriately.

(b) The symbol " $\Rightarrow$ " means *implies*. It connects sentences or logical statements; it does not connect values or expressions. For instance:

$$\sin^2 x + \cos^2 x = 1 \;\Rightarrow\; 2\cos^2 x + 3\sin^2 x = 2 + \sin^2 x\,.$$

The symbol " $\to$ " means *tends to*, and is used to denote limits.

(c) Use of parentheses. To multiply x by $-y$, we write $x(-y)$. "$x \cdot -y$" or "$x \times -y$" is incorrect, even if you reduce the size of the minus sign, raise it and stick it very close to y.
Also, $x - y$ and $x + (-y)$ are correct. "$x + -y$" is not, even if you reduce the size of the minus sign, raise it and stick it very close to y.
The derivative sign has precedence over the product. Therefore, $\dfrac{d}{dt} f(t)g(t)$ means one thing (namely, $g(t)\, df(t)/dt$) and $\dfrac{d}{dt}\big(f(t)g(t)\big)$ means another.

(d) We never use the integral symbol without the differential symbol. $\int f(x)$ is incorrect. If you are integrating with respect to x, you must write $\int f(x)\,dx$.

(e) The notations $\dfrac{dz}{dx}, \dfrac{\partial z}{\partial x}$ mean different things. The first is a total derivative (z is a function of x only, possibly through intermediate variables), whereas the second is a partial derivative.

[2] "$\sqrt{x^2} = x$" is wrong, since x may be negative. The correct formula is $\sqrt{x^2} = |\,x\,|$, which is more complete than "$\sqrt{x^2} = \pm x$".

[3] Polynomials are series, but not all series are polynomials.

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} \qquad \text{and}$$

$$1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} \qquad (1)$$

are respectively polynomials of degrees 3 and n, but

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots \qquad \text{and}$$

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots + \frac{x^n}{n!} + \ldots \qquad (2)$$

are bona fide infinite series, the second written less ambiguously than the first. The first occurence of three dots in (1) and (2) signify that terms extend up to order n . The second occurence of $\ldots$ in (2) means that the series does not stop at order n, making n simply a dummy variable. (For those of you who learned computer programming, a dummy variable is the same as a local variable). As you see, three dots make a big difference.

[4] A scalar expression is never comparable to a differential expression. The following are meaningless:

$$x + 3y = 4\,dx \qquad \text{equating scalar and differential}$$

$$x + 3y\,dy = 4\,dx \qquad \text{adding scalar and differential}$$

[5] When asked to verify that a given function solves a differential equation, simply verify (i.e.: plug in, substitute). The poorer strategy is to find the general solution of the differential equation, then check that the given function is indeed of that form. See, for example, all the exercises of § 1.2.

[6] Solving nonexact linear equations. It is preferable to break down the steps, showing along the way that the integrating factor indeed reduces the equation to exact form. For example,

$$\frac{dy}{dx} + 2y + e^{-3x} = 0\,.$$

First write in standard form:

$$\frac{dy}{dx} + 2y = -e^{-3x} \qquad (1)$$

then find the integrating factor:

$$\mu(x) = \exp\left(\int 2\,dx\right) = e^{2x}\,.$$

At this point, it is bad style to express the solution in one swoop. Do not write this:

$$y = e^{-2x}\left(\int -e^{-3x}\,dx\right)$$

but rather, display the form as an exact derivative resulting from multiplying (1) by the integrating factor:

$$e^{2x}\frac{dy}{dx} + e^{2x}\,y = -e^{-x}$$

$$\boxed{\frac{d}{dx}(e^{2x}y) = -e^{-x}} \qquad \textbf{exact derivative form} \qquad (2)$$

(this allows you to check that the integrating factor is indeed the correct one)

$$e^{2x}y = \int -e^{-x}\,dx$$

etc… Points will be taken off if equation (2) above is missing.

[7] In word problems, I do not use any symbol without first defining it. So if I am going to use the symbol x or V, where x or V is not part of the problem statement, I must state: “let x be the amount of lead in kgs” or “let V be the amount of acid in mL” …

In addition, the solution of word problems requires the use of verbal explanations throughout the solution, and this means complete sentences. You may write in Spanish. Most of you are not using any words, let alone complete sentences.

[8] (Style). In word problems, do not use units in front of every term of an equation. The following is incorrect:

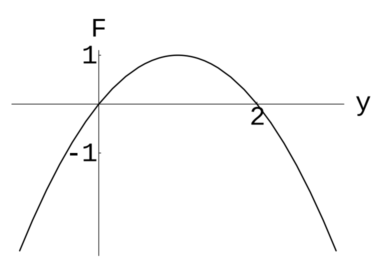
$$\frac{dx}{dt} = 4L/\text{min}(0.2kg/L) - (4L/\text{min})\frac{x}{100}\,.$$

Rather, write:

$$\frac{dx}{dt} = 0.8 - 4\frac{x}{100} \qquad (kg/\text{min})$$

The textbook does use the style I am asking you to avoid, in an explanatory fashion, in equations (2) and (3) on p. 97, (and again in example 2 of the following page), but foregoes it as soon as the mathematical equations are stated, starting from equation (4).

[9] At this stage of your education, you have taken and passed (or sought exemption from) precalculus and courses in calculus. We therefore expect evidence of analytical ability. In particular, you should be able quickly to draw the graph of low-order polynomials. When asked to sketch the direction field of, say, $dy/dt = -y(y-2)$, a sketch of this figure (note that the right-hand-side $F(y)$ is already given in factored form):



is quicker to obtain, and a lot more useful, than tabulating values like this:

y	$F(y)$
-1	-3
0	0
1	1
2	0
…	etc …

Present the first, not the second. If you need to tabulate, do it on scratch paper, but do not show the table.

[10] When applying the method of separation of variables to a first order equation, you will usually get, upon integration, a one-parameter family of solutions. But if, in the process of separating, you divided by a certain function of the dependent variable, then setting that function to zero will correspond to a solution of the differential equation not given by the one-parameter family. So you must examine this solution separately, by plugging directly in the differential equation. Missing this step means your solution is incomplete. For example, the equation

$$\frac{dy}{dt} = \sqrt{y}$$

gives, upon separating and integrating, the one-parameter family $y = (x + C)^2/4$. But in order to separate, one must divide by $\sqrt{y}$, and setting $\sqrt{y} = 0$ gives the solution $y = 0$ which does not correspond to any value of the parameter C .

In addition, a complete sequence of steps, when separating equations, consists of specifying the sign of the constants collected upon exponentiation (when applicable) and relating them to the missing solution (see steps (6), (7), (8) of example 2 p. 43). Putting together this example, the discussion on pp. 44–45, and exercise 30 p. 47, you see that this method is a little tricky. Should you use it as the preferred method when solving exam problems? Read notes [11] and [12].

[11] This is not a correction, but a substitution method, (in addition to the two which we studied in class), which you are required to know. It relies on the following formula, which is easy enough to obtain, and which you are also required to know:

$$\frac{dz}{dt} = kz \text{ has general solution } z = e^{kt} .$$

The substitution method applies to equations of the form

$$\frac{dy}{dt} = ay + b$$

and consists of making simply $z = ay + b$. Then the resulting equation in z becomes

$$\frac{dz}{dt} = a\,\frac{dy}{dt} = az$$

with solution $z = Ce^{at}$, and the solution in y is obtained by making $y = (z - b)/a$. This substitution applies to some models seen in § 3.2 – 3.4:

$$\frac{dx}{dt} = a - \frac{bx}{V} \qquad \text{mixing with fixed volume}$$

$$\frac{dT}{dt} = -K(T - M) \qquad \text{cooling without heat sources}$$

$$m\frac{dv}{dt} = mg - bv \qquad \text{falling with constant friction}$$

Identify those examples where this substitution applies, and redo them using it. Do the same for the practice exercises: in each of sections 3.2–3.4, there are problems where this substitution applies.

[12] A pointer about exam problems: when the same equation allows different methods of solution, your choice of solution, ranging from the most to the least efficient, will be reflected in your score. So, if you are solving, say,

$$\frac{dT}{dt} = -K(T - M) \qquad (2),$$

and the solution is worth 10 pts (which means, in effect, that I usually grade it over 12 pts), then you will get 12 pts for using the substitution of note [11] correctly, or 10 pts for regarding (2) as a linear equation and using integrating factor, or 8 pts for solving it as a separable equation (assuming you do not make mistakes).