Tips and common corrections

[1] Some comments about notation.

(a) The symbol " = " means *equals*, and nothing else. Points are lost on tests and quizzes for using it inappropriately.

(b) The symbol " $\Rightarrow$ " means *implies.* It connects sentences or logical statements; it does not connect values or expressions. For instance:

$$\sin^2 x + \cos^2 x = 1 \;\Rightarrow\; 2\cos^2 x + 3\sin^2 x = 2 + \sin^2 x\,.$$

The symbol " $\to$ " means *tends to,* and we will only use it to denote limits.

(c) The symbol " $\in$ " means *belongs to.* It relates elements to sets. The symbol " $\subset$ " means *is included in.* It allows to compare between sets. The two symbols are not interchangeable.

(d) Use of parentheses.

To multiply x by $-y$, we write $x(-y)$. "$x \cdot -y$" or "$x \times -y$" is incorrect, even if you reduce the size of the minus sign, raise it and stick it very close to y.
Also, $x - y$ and $x + (-y)$ are correct. "$x + -y$" is not, even if you reduce the size of the minus sign, raise it and stick it very close to y.

The derivative sign has precedence over the product. Therefore, $\dfrac{d}{dt} f(t)g(t)$ means one thing (namely, $g(t)\,df(t)/dt$) and $\dfrac{d}{dt}\left(f(t)g(t) \right)$ means another.

(e) We never use the integral symbol without the differential symbol. $\int f(x)$ is incorrect. If you are integrating with respect to x, you must write $\int f(x)\,dx$.

(f) The notations $\dfrac{dz}{dx}, \dfrac{\partial z}{\partial x}$ mean different things. The first is a total derivative (z is a function of x only, possibly through intermediate variables), whereas the second is a partial derivative.

[2] "$\sqrt{x^2} = x$ " is wrong, since x may be negative. The correct formula is $\sqrt{x^2} = |\,x\,|$, which is more complete than "$\sqrt{x^2} = \pm x$". For the same reason, $y^{2/3} = x$ is equivalent to $y^2 = x^3$, (where y may be negative), but not to $y = x^{3/2}$, where y is never negative.

[3] There is no order between complex numbers, as there is between real numbers. Therefore, "$z < w$" or "$-1 < z < 1$" are absurdities. Note that the domain of convergence of a series defined in C is not an "interval", but a disc given by $|z - z_0| < R$, where R is a (possibly infinite) radius.

[4] In the complex plane, the equation $z^n = w$ has n distinct roots (solutions). Therefore, we do not use the symbol $\sqrt[n]{w}$, since we cannot distinguish any particular root. This holds even if $n = 2$: we cannot say that $\sqrt{w}$ is the "positive" solution of $z^2 = w$, since there is no positive

or negative among complex numbers (see [3]). The symbol $\sqrt{-1}$ is a quaint holdover, even though found in some texts. Use i instead.

[5] When solving exact differential equations, assume you found the potential function to be $F(x, y) = x^2 - 2y$. The family of solutions is then described, not by

$$\text{“ } F(x, y) = x^2 - 2y + C \text{ ”}$$

(not clear how to make a sentence including this and the word “solution”, since all this is saying is that the potential is determined up to a constant), but rather by

“the solutions are the curves $x^2 - 2y = C$”,

which describes explicitly the one-parameter family of solutions.

[6]

$$f(x) = g(x) \quad \text{and} \quad \int f(x)\,dx = \int g(x)\,dx$$

are not equivalent statements. For this reason, neither are

$$(1) \quad \frac{d\,(\mu(x)y)}{dx} = g(x) \qquad \text{and} \qquad (2) \quad \int \frac{d(\mu(x)y)}{dx}\,dx = \int g(x)\,dx\,.$$

(1) is what I call exact derivative form, not (2). When using the method of integrating factors, you are required to show step (1). As for (2), which is the next step, it is better to write

$$\mu(x)y = \int g(x)\,dx,$$

on the principle that, contrary to students’ belief, the fewer integral symbols used, the better.