MATE 4052 assignment 1

Without further indication, exercises refer to the book of Coleman.

1. Exercise 1.7.

2. Exercises 1.8, 1.9.

3. Exercise 1.10.

4. If (E, d) is a metric space, define

$$d'(x, y) = \frac{d(x, y)}{1 + d(x, y)}.$$

Prove that d' is also a distance in E.

5. Let A and B be subsets of a metric space E. Prove that $\overline{A \cap B} \subset \overline{A} \cap \overline{B}$ and that $A \cap \overline{B} \subset \overline{A \cap B}$ if A is open.

6. In a metric space E, we say that A is dense in B if $A \subset B \subset \overline{A}$. Show that if A is dense in S and S is dense in B, then A is dense in B.

7. In a metric space, show that if A is dense in S and B is open in S (meaning $B = S \cap U$, where U is open in E), then $B \subset \overline{A \cap B}$. (Use exercise 5).