MATE 4052 assignment 4

17. Exercise 1.28.

18. Exercise 1.29.

19. Provide the detail of the last step of the proof that the map

$$u \mapsto u^{-1}$$

Which sends an element $u \in \mathrm{Isom}(E; F)$ to u^{-1} is continuous:

$$\forall \varepsilon > 0 \ \exists \delta > 0 : \ \|u - u_0\| \le \delta \Rightarrow \|u^{-1} - u_0^{-1}\| \le \varepsilon .$$

20. Let E be the Banach space of sequences $(\xi_n)_{n \ge 0}$ of real numbers such that $\lim \xi_n = 0$, equipped with the sup norm. Let $(e_n)_{n \ge 0}$ be the canonical basis of E.

(a) Show that for each $x = (\xi_n) \in E$, the series $\sum_{n=0}^{\infty} \xi_n e_n$ is convergent and has sum x in E.

(b) Let u be a continuous linear form on E, and $\eta_n = u(e_n)$. Show that the series $\sum \eta_n$ converges, and that the norm of u is given by

$$\|u\| = \sum_{n=0}^{\infty} |\eta_n| .$$

Conclude that the topological dual E^* of E can be identified with the space $l^1(R)$ of summable sequences of real numbers, equipped with a norm which you will specify.