MATE 4052 assignment 6

21–23. Exercises 3.2, 3.3, 3.4.

24. Classical mean-value theorem. In (a) and (b), f is a continuous real-valued function defined on the segment $[a, b]$, differentiable in (a, b).

(a) (Rolle's theorem). Assume $f(a) = f(b) = 0$. Show that there exists $c \in (a, b)$ such that $f'(c) = 0$.

(b) Show that in general (without the assumption of (a)), there exists $c \in (a, b)$ such that

$$f(b) - f(a) = f'(c)(b - a).$$

(c) Show that if $a, b \in R$, $a \neq b$, there exists no real number c such that

$$e^{ib} - e^{ia} = ie^{ic}(b - a).$$

This shows that the classical mean-value theorem does not apply to vector-valued functions.