MATE 4052 assignment 7

Not to be handed in.

Exercises 8.1–8.4.

Exercise 4.4.

(#) Let f be defined for all real x, and suppose that

$$|f(x) - f(y)| \leq (x - y)^2$$

for all real x and y. Prove that f is constant.

(#) Assume f' is continuous on $[a, b]$ and $\varepsilon > 0$. Prove that there exists $\eta > 0$ such that

$$\left| \frac{f(x) - f(y)}{x - y} - f'(x) \right| \leq \varepsilon$$

whenever $0 < |x - y| \leq \eta,\ a \leq x \leq b,\ a \leq y \leq b$.

(#) E, F are Banach spaces. Let U be open in E and $f : U \longmapsto F$. Assume f is differentiable for all $x \neq a$ (x, a in U, a fixed) and that $x \longmapsto f'(x)$ has a limit as x tends to a. Show that f is strictly differentiable at a, and that

$$\lim_{\substack{x \to a \\ x \neq a}} f'(x) = f'(a).$$