MATE 4071 problems

1–5: 14/08

1. Problem 12 of § 6.3 of Boas.

2. Problem 13 of § 6.3 of Boas.

3. Problem 14 of § 6.3 of Boas.

4. Problem 15 of § 6.3 of Boas. Improved hint (27/10): for the law of reflection, consider the unit vectors u, u_1 as given, and express that u_3 belongs to the plane they determine, by writing it as a linear combination of the two vectors u, u_1, with coefficients to be determined. The angle which u_1 makes with u will not figure explicitly in the combination.

5. Problem 16 of § 6.3 of Boas.

6–10: 18/08

6. (Do Carmo, Differential Geometry of curves and surfaces). Show that the equation of a plane passing through three noncolinear points p_1, p_2, p_3 is

$$(p - p_3) \cdot \big((p - p_1) \times (p - p_2)\big) = 0 \qquad (1)\,.$$

   You will do this in two steps:

   (i) A point p belonging to the plane passing through the three given points satisfies (1).

   (ii) If a point p in space satisfies (1), it belongs to the plane in question.

   For (i), use the fact that any point p belonging to the plane defined by p_1, p_2, p_3 is of the form $p = \alpha p_1 + \beta p_2 + \gamma p_3$, with $\alpha + \beta + \gamma = 1$. Let also $u = p_2 - p_1$ and $v = p_3 - p_1$ (draw the figure). Express then the left-hand side of (1) in terms of u, v, α, β, γ and verify that it is identically zero.

   For (ii), use (without proving) the fact that any point p in space is of the form $q + w$, where q belongs to the plane $p_1p_2p_3$ (in particular, q satisfies (1)) and w is a vector orthogonal to that plane – in other words, w is perpendicular to both u and v. Show then that, if p satisfies (1), w is necessarily the zero vector.

7. Problem 4 of § 6.4.

8. Problem 5 of § 6.4.

9. Problem 6 of § 6.4.

10. Problem 9 of § 6.4.

11–16: 28/08

11. Problem 7 of § 6.6.

12. Problem 9 of § 6.6.

13. Problem 11 of § 6.6.

14. Problem 13 of § 6.6.

15. Problem 14 of § 6.6.

16. Problem 17 of § 6.6: omit the z term in formula (6.7), and read the explanation on pp. 287–288.

17–21: 4/09

17. Choose 3 problems from 1–8 of § 6.7: one odd-numbered, two even-numbered.

18. Choose 3 problems from 9–16 of § 6.7: one odd-numbered, two even-numbered.

19. Problem 18 of § 6.7.

20. Problem 17 of the same section: (b), (c), (d).

21. Problem 17: (e), (j), and one of (g), (h), (i), (k).

22–27 : 11/09

22. Problem 4 of § 6.8.

23. Problem 7 of § 6.8: choose one of (a)-(b), one of (c)-(d).

24. Choose two of problems 8–15.

25. Problem 17 of § 6.8.

26. Problem 20 of § 6.8.

27. Problem 21 of § 6.8.

17/09: 28–31

28. Problem 6 of § 6.9.

29. Problems 7, 8 of § 6.9.

30. Problem 9 of § 6.9.

31. Choose one of pbs 10–12, § 6.9.

25/09: 32–40

32. Problems 1, 2 of § 6.10. You may use the formula for the surface of the sphere.

33. Choose two out of pbs 3–8.

34. Problems, 9, 10 of § 6.10. See problem 32.

35. Choose one of pbs 1–6, and one of pbs 7–10 § 7.2.

36. Problem 13 of the same section.

37. Problem 14.

38. Problem 8 § 7.3.

39. Choose two of problems 3–12 of § 7.4.

40. Problem 13, and one of pbs 14–16 of § 7.4.

30/09: 41-42

41. Problems 4, 6, 8 of § 7.5.

42. Using the definition of even and odd function, show that an even periodic function has a Fourier expansion in cosines only (including the constant term), and that an odd function has a Fourier expansion in sines only.

7/10: 43–46

43. Problem 1–11 of § 7.6: choose three of those functions.

44. Problems 14, 15 of § 7.6.

45. Problem 1–11 of § 7.7: choose three, not eleven.

46. Problem 13 of § 7.7.

14/10: 47–52

47. 1–9 of § 7.8, but choose two only.

48. Choose one of 11–14, and one of 16–20.

49. Problem 10 of § 7.9.

50. Problem 19 of the same section.

51. Problem 23.

52. Problem 24.

20/10: 53–56

53. Problem 4 of § 7.10. In this and the next exercise, make use of any symmetries.

54. Problem 2 of § 7.11.

55. (a)-(b): problems 6, 8 of § 7.11.

56. Problem 10 of § 7.11.

Posted 1/11: 57–62. In the exercises of § 7.12, use the definition of the Fourier transform given in class.

57. Problem 12 of § 7.12.

58. (a) - (b): problems 3, 17. In problem 3, take the support of f to be $[-1,1]$ instead of $[-\pi,\pi]$.

59. Problem 23.

60. Problem 24.

61. Choose one of problems 27–30.

62. Problem 31.

Posted 12/11: 63–65

63. (a)–(c): problems 2,4,6 of § 11.3.

64. (a)–(b): problems 12, 13 of the same section. Numerical evaluation is optional.

65. Problem 17.

Posted 18/11: 66–72

66. Problem 2 of § 11.5.

67. (a)–(c): Problems 3, 4, 6 of § 11.5.

68. (a)–(c): choose three of problems 1–8 of § 11.7, one odd and two even.

69. Problem 9.

70. (a)–(b): problems 10, 12 of § 11.7.

71. Problem 1 of § 11.8.

72. Problem 3 of the same section.

Posted 25/11: 73–82

73. Problem 1 of § 11.8.

74. Problem 2 of the same section.

75. Problem 3 of the same section.

76. Problem 2 of § 11.9.

77. Problem 5 of § 11.10. This problem refers to pbs 2 and 4.

78. Problem 6 of the same section.

79. Problem 1 of § 11.11.

80. Problem 2 of the same section.

81. Problem 3.

82. (a)–(b): problems 4, 5 of § 11.11.

Posted 4/12: not to be handed in. From § 11.12: pbs 1, 2. Choose three from pbs 4–13. Problems 17, 21, 22.

I also recommend that you try some of the problems entitled “Miscellaneous problems” from each of the chapters that we studied.