MATE 4072 problems

1. Problem 2 of § 13.2 of Boas.
2. Problem 11 of § 13.2 of Boas.
3. Problem 13 of § 13.2 of Boas.
4. Problem 14 of § 13.2 of Boas.
5. Problem 16 of § 13.2 of Boas.
6. Problem 1 of § 13.3 of Boas.
7. Problem 3 of § 13.3 of Boas.
8. Problem 7 of § 13.3 of Boas.
9. Using the d'Alembert formula, write the equation of the solution u of the one-dimensional wave equation

$$u_{tt} - u_{xx} = 0, \qquad u(0,x) = h(x), \quad u_t(0,x) = 0, \qquad (1)$$

where $h(x) = 1, -1 \le x \le 1, \quad h(x) = 0$ otherwise. Sketch the graph of $u(t, \cdot)$ at different times. (At time $t = 0$, the graph of u is the same as the graph of h.) At what time does the support of u (set of x-values where $u \neq 0$) split into two intervals? Hint: the two cones of influence originating from the points $(0, -1), (0, 1)$ divide the half-plane $t \ge 0$ into six regions. The solution will have a different equation in each of these regions.

10. Same as the previous problem, but with the data:

$$u(0,x) = 0, \qquad u_t(0,x) = h(x)$$

where h is the same function as above.

11. Indicate how to combine the solutions of the two previous problems to solve the wave equation (1), where the initial data is now

$$u(0,x) = h(x), \qquad u_t(0,x) = h(x)$$

12. a)–b): problems 22, 24 of § 14.2 of Boas.
13. a)–c): problems 36, 41, 42 of § 14.2 of Boas.
14. Problem 45 of § 14.2 of Boas.
15. Problem 46 of § 14.2 of Boas.
16. Problem 47 of § 14.2 of Boas.
17. a)–b): problems 51, 53 of § 14.2 of Boas.

18. a)–b): problems 60, 62 of § 14.2 of Boas.
19. Problem 3 of § 14.3 of the text.
20. Problem 4 of § 14.3 of the text.
21. Problem 11 of § 14.3 of the text.
22. Problem 15 of § 14.3 of the text.
23. Problem 19 of § 14.3 of the text.
24. Problem 24 of § 14.3 of the text.
25. a)–b): problems 7,8 of § 14.4.
26. Problem 11 of § 14.4.
27. Problem 1 of § 14.5.
28. a)–b): problems 6,7 of § 14.6.
29. Problem 12 of § 14.6.
30. a)–c): problems 17, 25, 34 of § 14.6.
31. a)–c): problems 30, 31, 32 of § 14.6.
32. a)–b): problems 4, 9 of § 14.7.
33. a)–b): problems 3, 20 of § 14.7.
34. Problem 25 of § 14.7. For example 4, we used the Jordan inequality.
35. Problem 26 of § 14.7. See previous comment.
36. Problem 41 of § 14.7.
37. Problem 42 of § 14.7.
38. a)–b): problems 1, 10 of § 15.1.
39. a)–b): problems 6, 9 of § 15.1.
40. a)–b): problems 1, 6 of § 15.2.
41. Problem 13 of § 15.2.
42. Problem 17 of § 15.2.
43. Problem 18 of § 15.2.
44. Problem 19 of § 15.2.

45. Problem 10 of § 15.3.

46. Problem 15 of § 15.3.

47. Problem 16 of § 15.3.

48. Problem 17 of § 15.3.

49. Problem 20 of § 15.3.

50. Referring to the coin tossing game of problem 21 of § 15.3, assume that the winner is the first player to repeat the outcome of his last toss. So, if we code the moves by listing the outcomes in sequence (*tth* means player one tosses tails, player two tosses tails, player one tosses heads…), the outcome *httht* means that player one wins (tails on his third toss repeats the result of his second toss) and the outcome *httt* means that player two wins (tails on his second toss repeats the result of his first toss). For this version of the game, using a suitable representation as a finite-state Markov chain (see example from lecture), find the probability that player one wins.

51. Problem 4 of § 15.4.

52. Problem 6 of § 15.4.

53. Problem 9 of § 15.4.

54. Problem 22 of § 15.4.

55. Problem 4, § 15.5. Replace “tops” by “bottoms”, since gravity on Mars has the same effect as on earth.

56. Problem 7, § 15.5.

57. Problem 11, § 15.5. Use X, Y, keeping with our use of capital letters to denote random variables.

58. a)–b): problems 8, 15 of § 15.5.

59. Problem 1 (b),(c) of § 15.6. Part (a) was shown in class.

60. Problem 2, § 15.6.

61. Problem 6, § 15.6.

62. Problem 3, § 15.8.

63. a)–c): problems 13, 17, 19 of § 15.8. Use only the normal approximation. If X is binomially distributed, $P(X = k)$ is approximated, using the normal distribution, by $P(k - 1/2 < X < k + 1/2)$.

64. Problem 5 of exam 2.

65. Problem 2 of § 15.9.

66. a)–b) problems 6, 9 of § 15.9.

67. Problem 10 of § 15.9.

68. Problem 11 of § 15.9.

69. Problem 1 of § 15.11.

70. Problem 2 of § 15.11.

71. Problem 4 of § 15.11.

72. Problem 7 of § 15.11.

73. Problem 9 of § 15.11.

74. Problem 10 of § 15.11. Use only the normal approximation.

75. Problem 16 of § 15.11.