MATE 4145 First Examination September 29, 2016

Name:

1. Let

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 3 & 0 \\ 1 & 5 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

(a) Find the values of λ such that $\det(\lambda A - B) = 0$.

(b) Find all solutions x of the system $Ax = Bx$ ($x \in R^3$).

2. Use the direction field plot to sketch the solution of the initial-value problem

$$\frac{dx}{dt} = x(x-1), \qquad x(0) = 0.5$$

in the (t, x) plane.

3.

(a)

$$A = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}.$$

Find all matrices C such that both $AC = CA$, $BC = CB$, hold.

(b) Can the vector $z = (2, 3, 2)$ be written as

$$z = \alpha x + \beta y$$

where $x = (2, 3, 0)$ and $y = (1, -1, 1)$?

4. Is either of these transformations linear? Justify

(a) $T(x_1, x_2, x_3) = (x_2 - x_1^2, 2x_3)$.

(b) $T(x_1, x_2) = x_1 - x_2 + 2$.

5. $S : R^2 \mapsto R^3$ is defined by

$$S(x_1, x_2) = (x_1 - x_2, x_2, 2x_1)$$

and $T : R^3 \mapsto R^2$ is defined by

$$T(x_1, x_2, x_3) = (x_3 - x_1 + x_2, x_2 + 2x_3).$$

Find the matrix which is associated with the composition $S \circ T$.