MATE 4145 First Examination October 3, 2019

Name:

1. (20) For which values of a, b is the matrix

$$A = \begin{pmatrix} 1 & a & 0 \\ 0 & 2 & b \\ 0 & 0 & 1 \end{pmatrix}$$

invertible? Find the inverse, for those values.

2. (20)

(a) Is either of these transformations linear? Justify

$T(x_1, x_2, x_3) = (x_2 - x_1^2, 2x_3)$.

$T(x_1, x_2) = x_1 - x_2 + 2$.

(b) Let A be an $n \times n$ matrix. Assume the only solution to $Ax = 0$ is $x = 0$. Can one find a vector b such that the equation $Ax = b$ has no solution? Explain.

3. (30) Let $A = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 2 & 0 \\ 2 & 0 & -1 \end{pmatrix}$. Find the elementary matrices $E_1, \ldots E_5$ such that

$$E_5 E_4 \cdots E_1 A = I,$$

and using this (and no other method) find $\det(A)$.

4. (30)

(a) Plotting the function $x + x^2$, study the stability of the equilibriæ of the differential equation

$$x' = x + x^2 .$$

For which values of λ is $x = 0$ a stable equilibrium of the equation $x' = \lambda(x + x^2)$?

(b) Using isoclines, sketch the direction field of the equation

$$x' = 2x + t .$$

Find the isocline which is also a solution curve (state its equation), draw it, and draw some solutions on each side of it.