MATE 5049 assignment 1

1.

a) Show that if, for all $\lambda \in R$, $a\lambda^2 + 2b\lambda + c \geq 0$, then $b^2 - ac \leq 0$. *Hint:* find the minimal value of the polynomial in λ.

b) Apply this to the expression $\sum_{i=1}^{n}(x_i - \lambda y_i)^2$ to obtain:

$$\sum_{i=1}^{n} x_i y_i \leq \left(\sum_{i=1}^{n} x_i^2 \right)^{\frac{1}{2}} \left(\sum_{i=1}^{n} y_i^2 \right)^{\frac{1}{2}}$$

for $x, y \in R^n$. This is called the *Cauchy-Schwarz inequality.*

c) Use this to show that the Euclidian distance in R^n satisfies the triangular inequality.

2. Let A be a subset of the normed space X, $\overset{o}{A}$ its interior. Show that $\overset{o}{A}$ is open.

3. Problem 3, p. 43 in Luenberger.

4. Problem 4, p. 43.

5. Problem 5, p. 43.