**MATE 6677 assignment 2**

7. Give an example of an essentially bounded function $f$ such that

$$\lim_{x \to 0} \| f_x - f \|_\infty = 0$$

does not hold.

8. As a preliminary step to proving theorem (0.13), we proved the lemma: If $\Phi \in L^1$ and $g(x) = \int k(x,y)\Phi(y)\,dy$, where $k(.,y) \in L^p$ $(1 < p < \infty)$, then

$$\|g\|_p \leq \int \|k(.,y)\|_p |\Phi(y)|\,dy$$

When $p > 1$, we used a characterisation of the p-norm in terms of the dual norm, and Fubini's theorem. Prove the lemma directly when $p = 1$.

9. We claimed that the function

$$f(t) = \begin{cases} e^{-\frac{1}{1-t^2}}, & -1 < t < 1 \\ 0, & |t| \geq 1 \end{cases}$$

is infinitely differentiable. As a step, show that it is $C^1$ at $t = 1$.

Marks: 6 + 6 + 12 = 24.