MATE 6677 assignment 4

The ambient space in all the problems of this set is R^n.

14. Let f, g be continuous functions on R^n. Using proof or counter-examples, relate in general the support of fg with the supports of f and g.

15. Find a sequence of test functions converging to $f = 1$ as distributions. (Note that $1 \notin \mathcal{D}$). Such a sequence exists, since $\mathcal{D}$ is dense in $\mathcal{D}$' for the topology of $\mathcal{D}$'.

16. Show that the inclusion of $\mathcal{D}$ into $\mathcal{D}'$ is "sequentially continuous". This amounts to showing that, if a sequence ϕ_j converges to zero in $\mathcal{D}$, then it also converges to zero in $\mathcal{D}$'.

17. Show that if u is a distribution and ϕ a test function (in R^n), then

$$f(x) = \langle u, (\tilde{\phi})_{-x} \rangle$$

is a continuous function of x.

18. Let u be a distribution of compact support, and $\psi \in \mathcal{D}$. Show that $u * \psi \in \mathcal{D}$.

19. Let u be a distribution of compact support. Show that if T is the operator on $\mathcal{D}$ defined by

$$T\psi = u * \psi,$$

then the transpose of T is $\tilde{u} * \psi$.

Marks: $6 + 6 + 6 + 9 + 9 + 6 = 42$.