MATE 6677 assignment 5

20. Verify that the assumptions of the Cauchy-Kovalevsky theorem stated in class are satisfied, and compute the terms of order at most two of the series expansion of the solution about the origin of the Cauchy problem

$$u_t = u^2 + u_x\,; \qquad u(0,x) = 1 + 2x\,.$$

21. Verify that the assumptions of the Cauchy-Kovalevsky theorem stated in class are satisfied, and compute the series expansion of the solution about the origin of the Cauchy problem

$$u_t = \sin u_x\,; \qquad u(0,x) = \frac{\pi}{4}x\,.$$

22. State the Cauchy-Kovalevsky theorem in enough generality (and credit your source) to be able to show that the Cauchy problem

$$u_{tt} = u_{xx} + u, \qquad u(0,x) = e^x, \quad u_t(0,x) = 0, \qquad (t,x) \in R \times R$$

has an unique analytic solution in a neighbourhood of $\{0\} \times R$. Find the solution in the form

$$u(t,x) = \sum_{k=0}^{\infty} a_k(x) t^k,$$

identifying the coefficients a_k. Examine also the convergence of this series.

23. The general solution of a partial differential equation of the first order in two independent variables involves one arbitrary function of one variable. Establish this result for the special equation

$$u_x + u_y = 0.$$

Hint: make a change of coördinates by rotating the xy frame by $\pi/4$.

24. Use the formula (0.1) of Folland to compute the normal derivative of the Newtonian potential (introduced in the proof of the mean value theorem) on the sphere of radius r centered at x, in dimensions $n = 2$ and $n > 2$.

Marks: $8 + 9 + 6 + 8 + 6 = 37$.