# MATE 6677 assignment 8

33. Formulate and prove Duhamel's principle for solving the initial value problem for the nonhomogeneous heat equation

$$\partial_t u - \partial_{xx} u = f(t,x), \qquad t > 0, \quad -\infty < x < \infty$$

$$u(0,x) = 0, \qquad -\infty < x < \infty$$

and apply the principle to find the solution:

$$u(t,x) = \int_0^t \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi(t-\tau)}} e^{-\frac{(x-\xi)^2}{4(t-\tau)}} f(\tau,\xi)\, d\xi\, d\tau$$

34.

a) Show that for $n = 3$, the general solution of the wave equation with spherical symmetry about the origin has the form

$$u = \frac{F(r+ct) + G(r-ct)}{r}, \qquad r = |x|$$

with suitable $F, G$.

b) Show that the solution with initial data of the form

$$u = 0, \quad \partial_t u = g(r)$$

($g$ even function of $r$) is given by

$$u = \frac{1}{2cr} \int_{r-ct}^{r+ct} \rho g(\rho)\, d\rho \qquad (1).$$

c) For

$$g(r) = \begin{cases} 1, & 0 < r < a \\ 0, & r > a \end{cases}$$

Find $u$ explicitly from (1) in the different regions bounded by the cones $r = a \pm ct$ in $tx$ space. Show that $u$ is discontinuous at $(0, a/c)$, due to focussing of the discontinuity of $\partial_t u$ at $t = 0, |x| = a$).

Marks: 9 + 12 = 21