MATE 6677 assignment 9

35. In the proof of Theorem (6.7), we used the fact that the two functions of ξ

$$(1 + |\xi|^2)^k, \qquad \sum_{|\alpha| \le k} (2\pi)^{|2\alpha|} |\xi^{2\alpha}| = \sum_{|\alpha| \le k} (2\pi)^{|2\alpha|} \xi^{2\alpha}$$

are asymptotically equivalent, meaning that each is bounded by a fixed positive scalar multiple of the other. Prove this claim.

36. (Rauch)

a) For which values of s is the characteristic function $\chi_{[0,1]}$ in $H_s(R)$?

b) For which values of s is $\chi_{[0,1]\times[0,1]}$ in $H_s(R^2)$?

37.

a) Show that the Dirac measure concentrated at zero is a tempered distribution on R^n.

b) Compute its Fourier transform.

c) For which values of s is δ in $H_s(R^n)$?

d) If K is the tempered distribution solution of $(I - \Delta)K = \delta$, for which values of s is K in $H_s(R^n)$?

38. According to corollary (6.8), the gradient of a function in H_1 is itself in L^2. Show that there exists no positive constant C such that

$$\int |u(x)|^2\, dx \le C \int |\nabla u(x)|^2\, dx, \qquad \forall u \in H_1(R^n)$$

39. Let $u \in H_s(R^n)$ with $s > n/2$. Show that $\lim_{x \to \infty} u(x) = 0$.

40. The product of two functions in $H_1(R)$ is also in $H_1(R)$. Hint: use the previous exercise.