ACTIVITY #1 – THREE-DIMENSIONAL CARTESIAN SPACE, FUNDAMENTAL PLANES AND SECTIONS

Name:_____

1) Draw each of the following points in three-dimensional space:

   a. $(3,4,2)$ b. $(-4,3,2)$

   c. $(2,1,-3)$ d. $(-3,0,3)$

2) In this problem to move "forward" or "backwards" is to move in the direction of positive *x* (the *x* coordinate increases, the *y* and *z* remain the same) or negative *x* (the *x* coordinate decreases, the *y* and *z* remain the same), respectively; "to the right" or "to the left" is moving in the positive or negative *y* direction, respectively; "up" or "down" is moving in the positive or negative *z* direction, respectively:

   a. Find the coordinates of the point where one ends if one starts at point $A(1,2,3)$ and moves 5 units forward, 4 units to the left, and 2 units up.

   b. Find the coordinate of the point where one ends if one starts at point $A(3,-4,2)$ and moves 4 units backwards, 4 units to the right, and 4 units down.

3) Draw all points in three-dimensional space that satisfy:

   a. $x=-2$ b. $y=4$

   c. $z=0$ d. $x=3$

4) Find an equation for each one of the coordinate planes:

   a. *xy* plane b. *xz* plane c. *yz* plane

5) Draw in three-dimensional space and describe symbolically the set of points that results from:

   a. intersecting the plane $x=1$ with the plane $y=2$ (the intersection consists **ONLY** of the points that are on both planes at the same time; draw **ONLY** those points, that is, do not draw the planes)
   b. intersecting the plane $y=-1$ with the plane $z=4$

6) In each of the following problems you are given a set *S* in three-dimensional space and a fundamental plane. The "intersection" of the plane and the set *S* consists of the points that are on the plane and that are also in the set *S*. For each of the problems in parts a, b, and c, follow all the instructions from i to iii.
   i. Draw the intersection on a Cartesian plane, identifying the axes. Note: to draw the intersection one does not need to draw or even know how the graph of set *S* looks.
   ii. Draw the intersection in three-dimensional space. Make sure that all the points in the graph are on the corresponding fundamental plane.
   iii. Find the coordinates of three of the points on the intersection.

   a. $S=\{(x,y,z): z=x^2+xy^2\}$; plane $x=1$

   b. $S=\{(x,y,z): z=x^2+(2+y)^3x+y^2\}$; plane $y=-2$

   c. $S=\{(x,y,z): z=x^2+y^2\}$; plane $z=4$

7) Draw each of the following sets in Cartesian three-dimensional space. To do so one doesn't need to draw or know how the graph of $z=xy^2$ looks. Each one of the sets is the intersection of a plane with the surface that is the graph of $z=xy^2$.

   a. $\{(x,y,z): z=xy^2, x=0\}$ Hint: It consists of more than one point.

   b. $\{(x,y,z): z=xy^2, x=1\}$

c. $\{(x,y,z): z = xy^2, x = 2\}$

d. $\{(x,y,z): z = xy^2, z = 1\}$

8) In each one of the following cases draw in three-dimensional space the intersection of the graph of $z = x\sin(y)$ with the given plane. To do the problem one does not need to draw or know how the graph of $z = x\sin(y)$ looks.

a. $x = 0$ (has more than one point)

b. $y = 0$ (has more than one point)

c. $z = 0$

d. $x = 1$

e. $x = 2$

f. $y = \pi / 2$

9) In each of the following problems draw in three-dimensional space and describe the intersection of set *S* with the given axis. There is no need to know the graph of *S* to do this.

a. $S = \{(x,y,z): z = x^2 + xy^2\}$ with the *y* axis. (It is contained in the *y* axis.)

b. $S = \{(x,y,z): z = x^2 + (2+y)^3 x + y^2\}$ with the *x* axis.

c. $S = \{(x,y,z): z = x\sin(y)\}$ with the *z* axis.

10) Let $S = \{(x,y,z): x^2 + x + y^2 = 2\}$. Do the following problems without drawing the graph of surface *S*.

a. Draw in three-dimensional space and describe the intersection of *S* with the *x* axis. Note that the intersection must be entirely contained within the *x* axis. On a separate drawing represent the intersection of *S* with the *xz* plane. Explain as carefully as you can why in both situations (*x* axis, *xz* plane) you obtain exactly the same equation but the answers are different.

b. Draw in three-dimensional space and describe the intersection of *S* with the *y* axis. On a separate drawing represent the intersection of *S* with the *yz* plane.

c. Draw in three-dimensional space and describe the intersection of *S* with the *z* axis. On a separate drawing represent the intersection of *S* with the *xy* plane.