Lecture 16: String Matching

CLRS- 32.1, 32.4

Outline of this Lecture

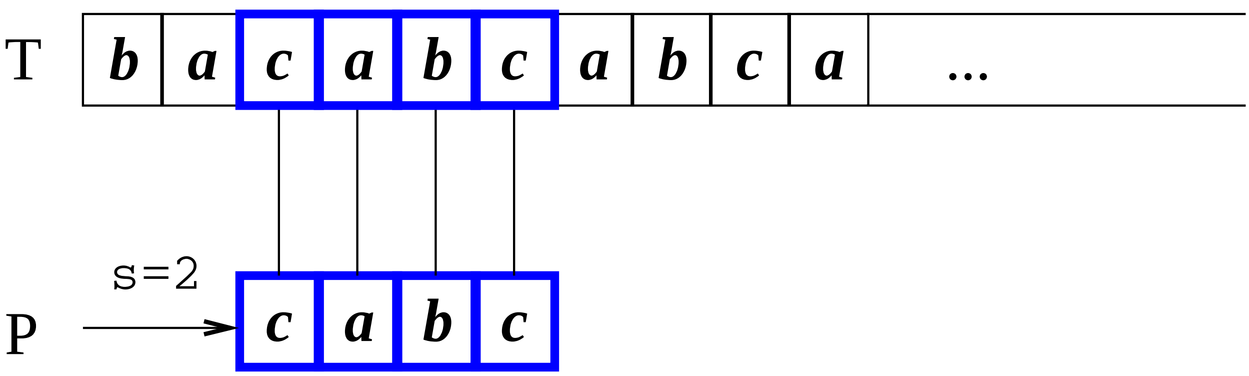
- String Matching Problem and Terminology.
- Brute Force Algorithm.
- The Knuth-Morris-Pratt (KMP) Algorithm.
- The Boyer-Moore (BM) Algorithm.

String Matching Problem and Terminology

Given a text array $T[1 \ldots n]$ and a pattern array $P[1 \ldots m]$ such that the elements of T and P are characters taken from alphabet Σ. e.g., $\Sigma = \{0, 1\}$ or $\Sigma = \{a, b, \ldots, z\}$.

The **String Matching Problem** is to find *all* the occurrence of P in T.

A pattern P occurs with **shift** s in T, if $P[1 \ldots m]$ = $T[s + 1 \ldots s + m]$. The String Matching Problem is to find all values of s. Obviously, we must have $0 \leq s \leq n - m$.



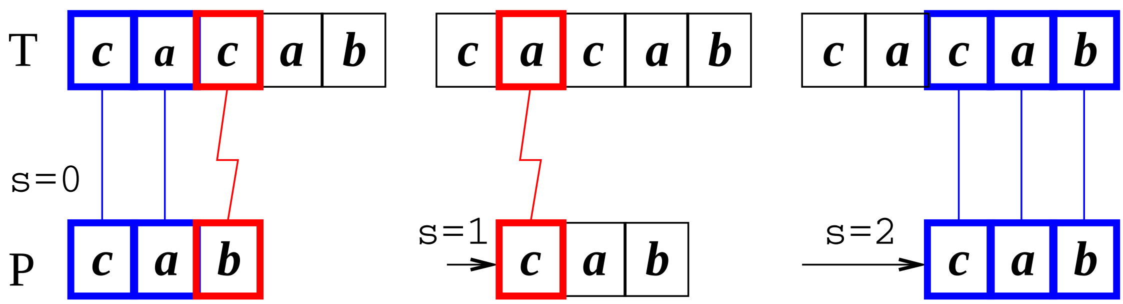
String Matching Problem and Terminology

A string w is a **prefix** of x if x =w y, for some string y.

Similarly, a string w is a **suffix** of x if x =yw , for some string y.

Brute Force Algorithm

Initially, P is aligned with T at the first index position. P is then compared with T from **left-to-right**. If a mismatch occurs, "slide" P to *right* by 1 position, and start the comparison again.



Brute Force Algorithm

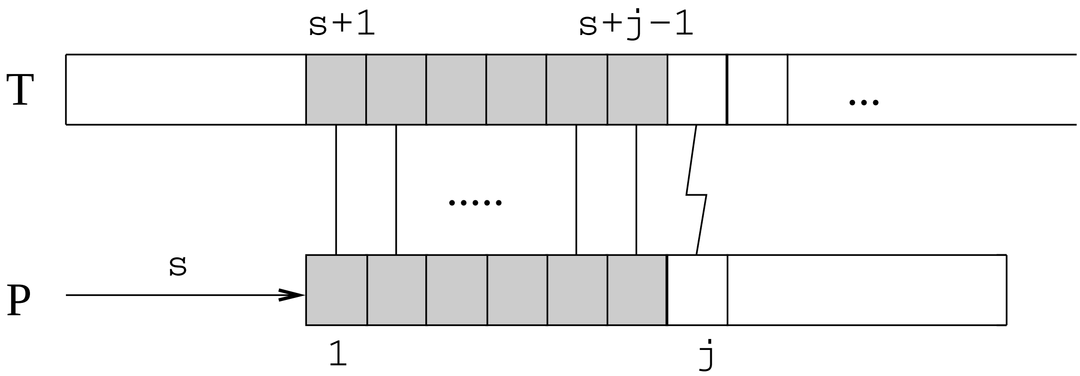
```
BF_StringMatcher(T, P) {
 n = length(T);
 m = length(P);

 // s increments by 1 in each iteration
 // => slide P to right by 1
 for (s=0; s<=n-m; s++) {
  // starts the comparison of P and T again
  i=1; j=1;
  while (j<=m && T[s+i]==P[j]) {
   // corresponds to compare P and T from
   // left-to-right
   i++; j++;
  }
  if (j==m+1)
   print "Pattern occurs with shift=", s
 }
}
```

The Knuth-Morris-Pratt (KMP) Algorithm

In the Brute-Force algorithm, if a mismatch occurs at $P[j]$ ($j > 1$), it only slides P to right by 1 step. It throws away one piece of information that we've already known. What is that piece of information ?

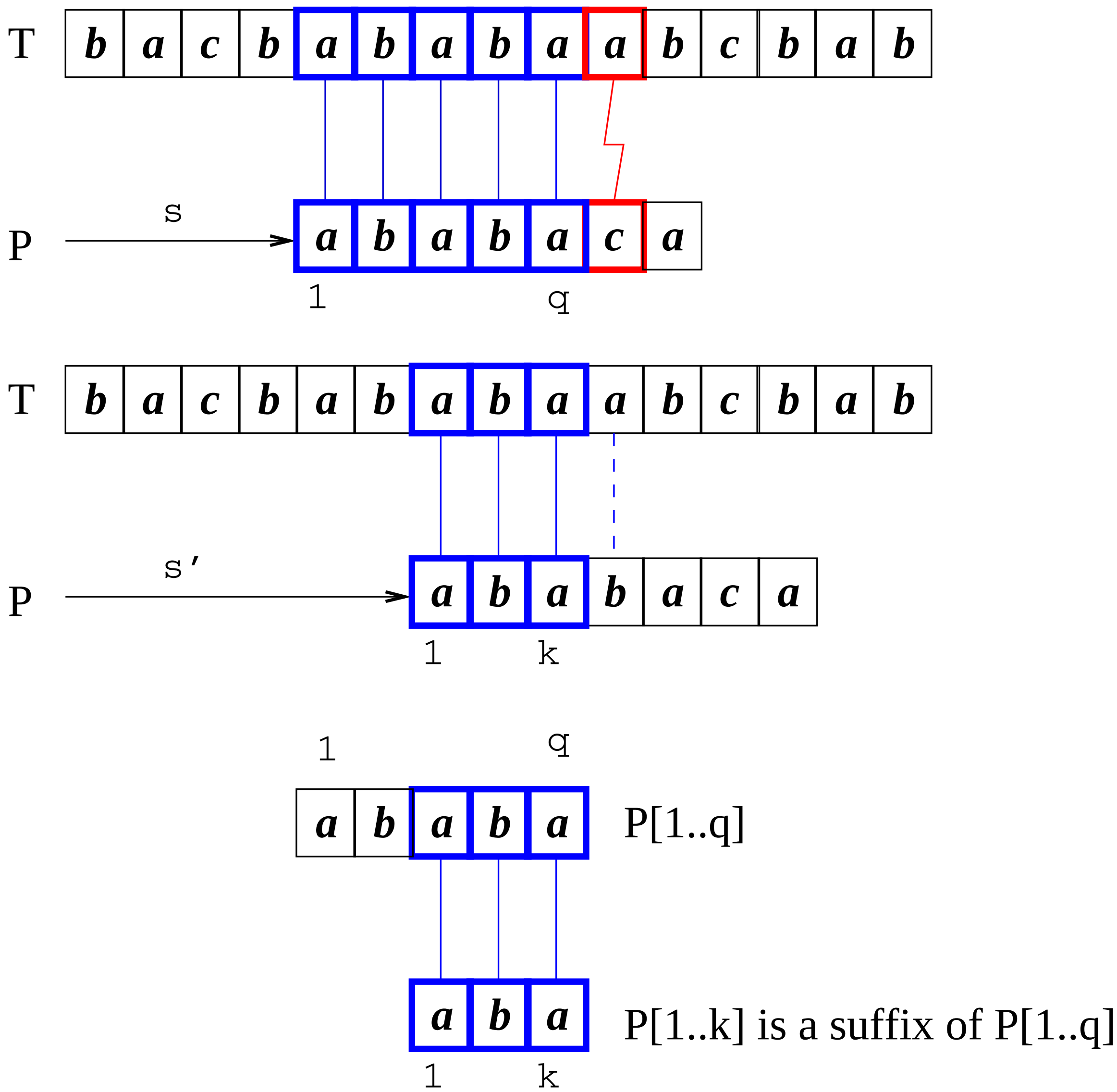
Let s be the current shift value. Since it is a mismatch at $P[j]$, we know $T[s+1..s+j-1] = P[1..j-1]$.



How can we make use of this information to make the next shift? In general, P should slide by $s' > s$ such that $P[1..k]$ = $T[s'+1..s'+k]$. We then compare $P[k+1]$ with $T[s'+k+1]$.

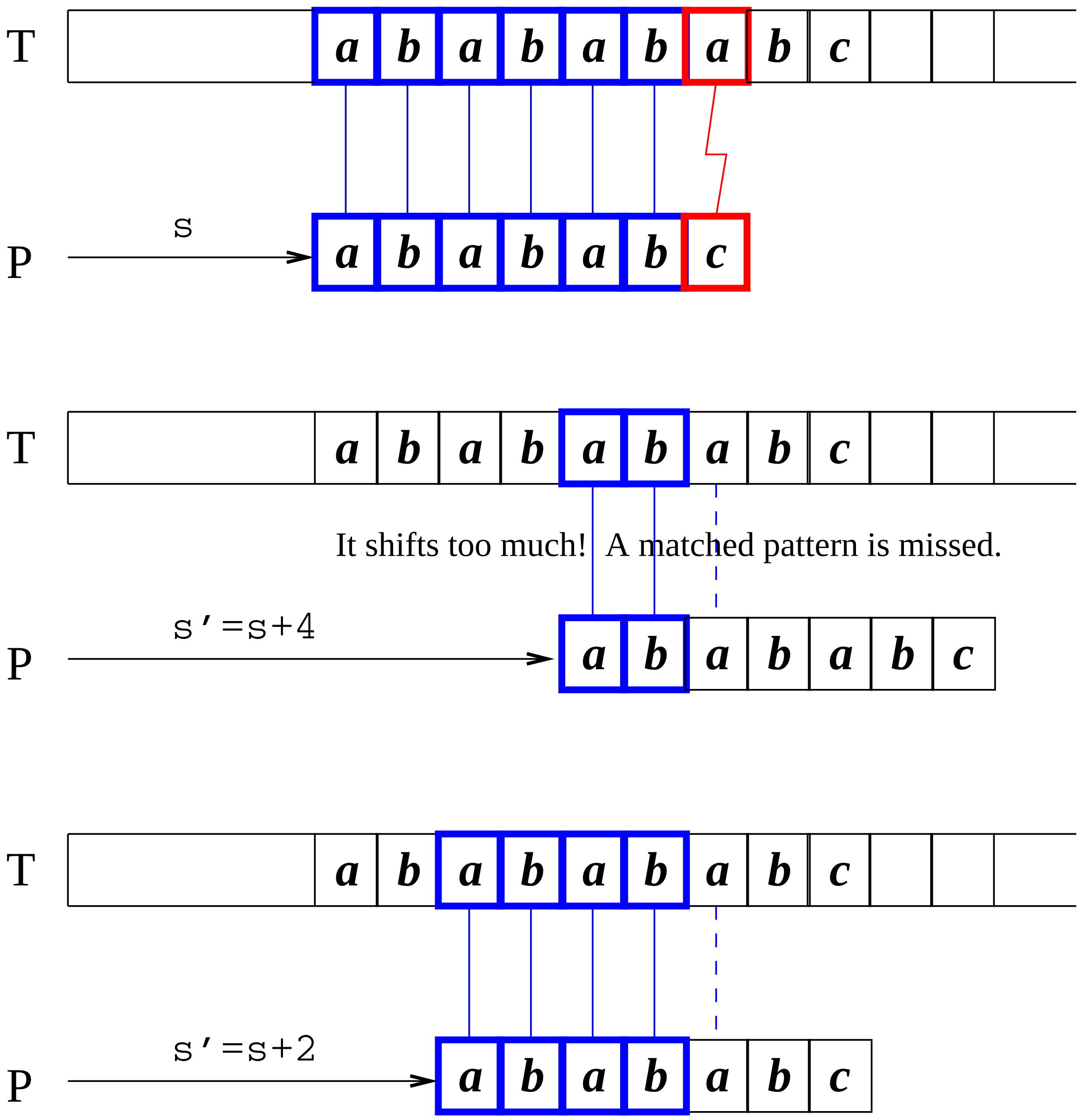
The Knuth-Morris-Pratt (KMP) Algorithm

When we slide P to right, it should be a place where P could possibly occur in T.



Do not shift too much

Do not shift too much, as it may miss some matched patterns!



The *next* function

We need to answer the following question: Given $P[1..q]$ match text characters $T[s+1..s+q]$, what is the *least* shift $s^{'} > s$ such that

$P[1..k]$ = $T[s^{'}+1..s^{'}+k]$,

where $s^{'}+k = s+q$?

In practice, the shift $s^{'}$ can be precomputed by comparing P against itself. Observe that $T[s^{'}+1..s^{'}+k]$ is a known text, and it is a **suffix** of $P[1..q]$. To find the *least shift* $s^{'} > s$, it is the same as finding the *largest* $k < q$, s.t.,

$P[1..k]$ is a suffix of $P[1..q]$.

The *next* function

Given $P[1..m]$, let *next* be a function $\{1, 2, \ldots, m\} \rightarrow \{0, 1, \ldots, m-1\}$ such that

next(q)$= max\{k : k < q$ and $P[1..k]$ is a suffix of $P[1..q]\}$.

q	1	2	3	4	5	6	7	8	9	10
P[q]	***a***	***b***	***a***	***b***	***a***	***b***	***a***	***b***	***c***	***a***
next(q)	0	0	1	2	3	4	5	6	0	1

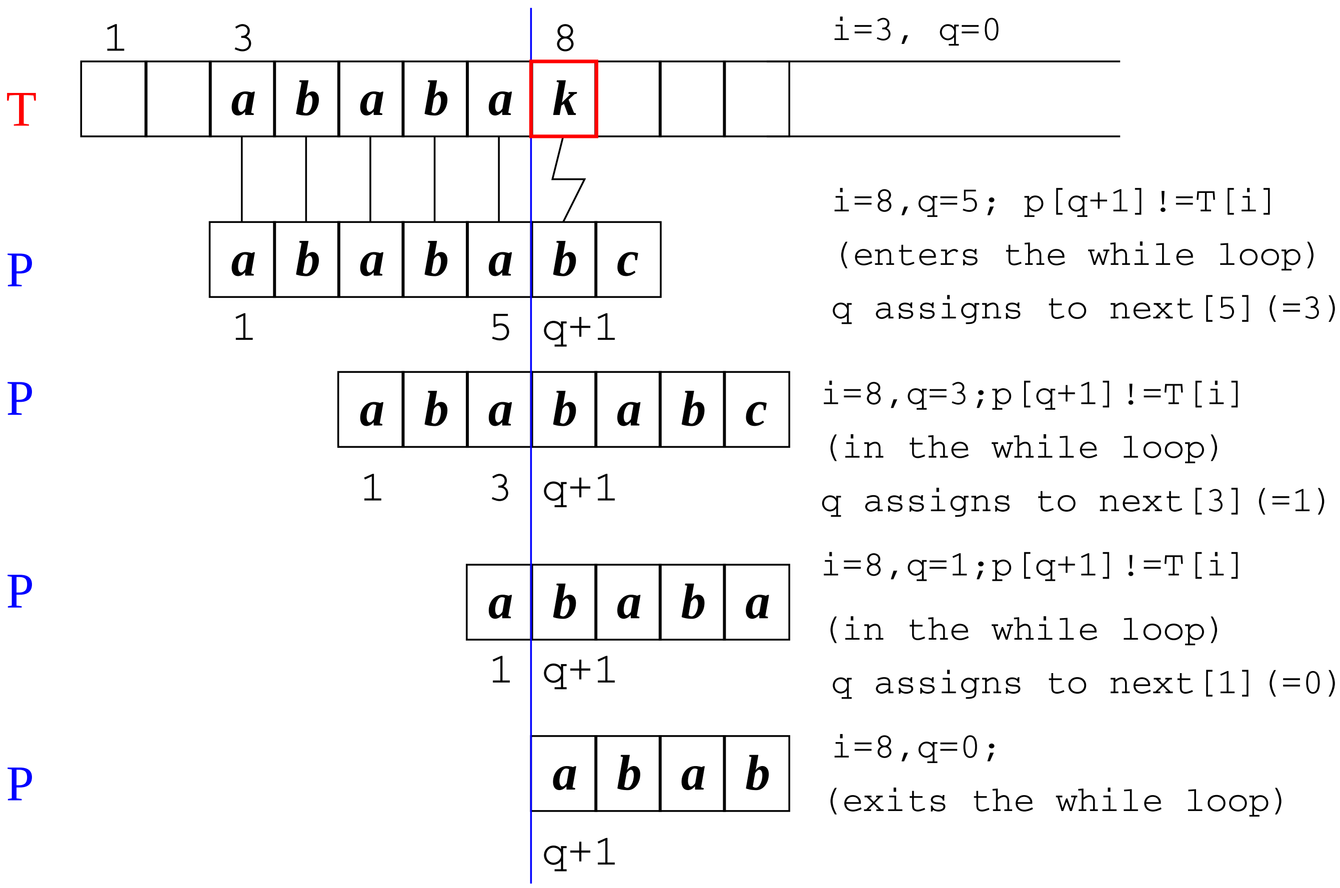
Given *next(q)* for all $1 \leq q \leq m$, we can use the KMP algorithm.

The Knuth-Morris-Pratt (KMP) Algorithm

```
KMP_StringMatcher(T, P) {
 n = length(T); m = length(P);
 compute_Next(P);
 q = 0; // number of characters matched
        // so far
 i=1;
 while (i<=n) {
  // loop until a match is found, or
  // number of characters matched so far
  // is 0; note 'i' is unchanged.
  while (q > 0 and P[q+1] != T[i]) {
   q=next[q];
  }
  // matched character increased by 1
  if (P[q+1]==T[i]) q=q+1;
  if (q==m) {
   print "Pattern occurs with shift=", i-m
   q=next[q];
  }
  i++;
 }
}
```

The Knuth-Morris-Pratt (KMP) Algorithm

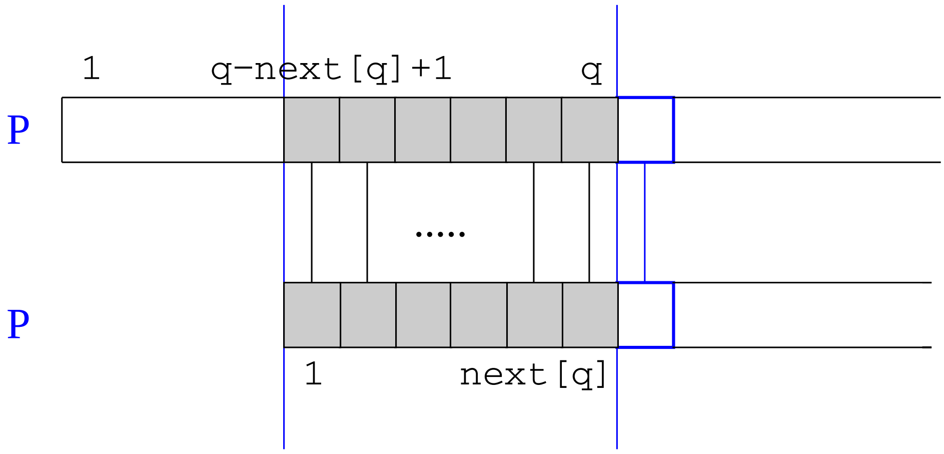
q	1	2	3	4	5	6	7
P[q]	***a***	***b***	***a***	***b***	***a***	***b***	***c***
next(q)	0	0	1	2	3	4	0



How to compute *next* function

Given `next[1], next[2], ..., next[q]`, how can we compute `next[q+1]`?

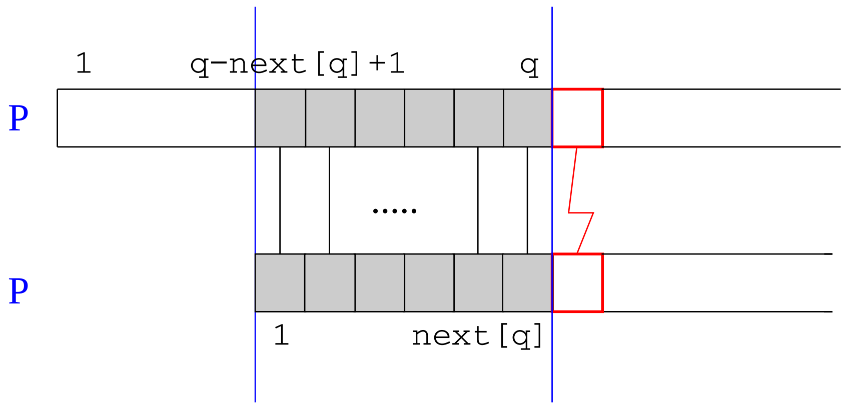
1. If `P[q+1]==P[next[q]+1]`, then `next[q+1]=next[q]+1`.



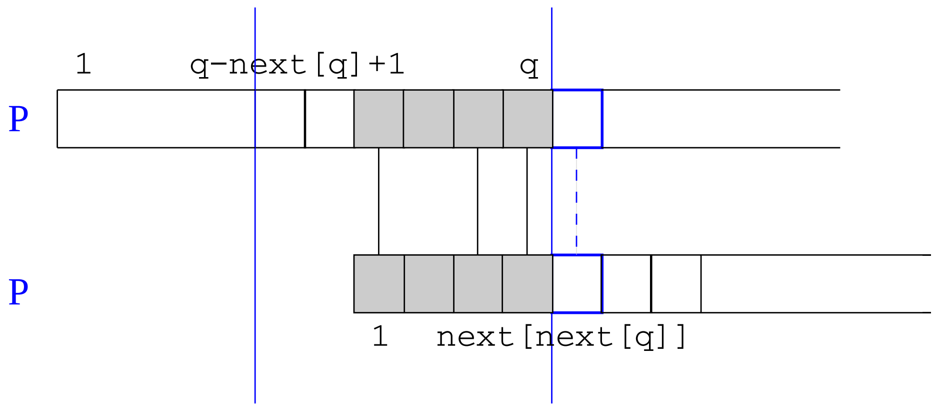
How to compute *next* function

2. If `P[q+1]!=P[next[q]+1]`, then do what?

   P should slide to a place such that the prefix of P `[1..next[q]]` occurs as a suffix of P `[q-next[q+1]..q]`; this information is stored in `next[next[q]]` !



`observe that P[1..next[q]]= P[q−next[q]+1..q]`



How to compute *next* function

We first set `next[1]=0`, then compute $next[q]$ with $q = 2, 3, \ldots m$, one by one in $m - 1$ iterations.

```
compute_Next(P) {
 m = length(P);
 next[1]=0; // initialization
 k = 0; // number of characters matched
        // so far
 q=2;
 while (q<=m) {
  while (k > 0 and P[k+1] != P[q]) {
   k = next[k];
  }
  if (P[k+1]==P[q]) k=k+1;
  next[q]=k;

  q++;
 }
}
```

Running Time of the KMP Algorithm

1. `compute_Next`

   (a) $3q - k = 6$ at the beginning, and $3q - k \leq 3m$ at all times.

   (b) Note that after each comparison, $3q - k$ increases *at least* by 1. But the value of $3q - k$ starts at 6, and the largest possible value is $3m$, it implies there are $O(m)$ number of comparisons.

   (c) Hence, the running time of `compute_Next` is $O(m)$.

Running Time of the KMP Algorithm

2. `KMP_StringMatcher`

   (a) $3i - q = 3$ at the beginning, and $3i - q \leq 3n$ at all times.

   (b) Note that after each comparison, $3i - q$ increases *at least* by 1.

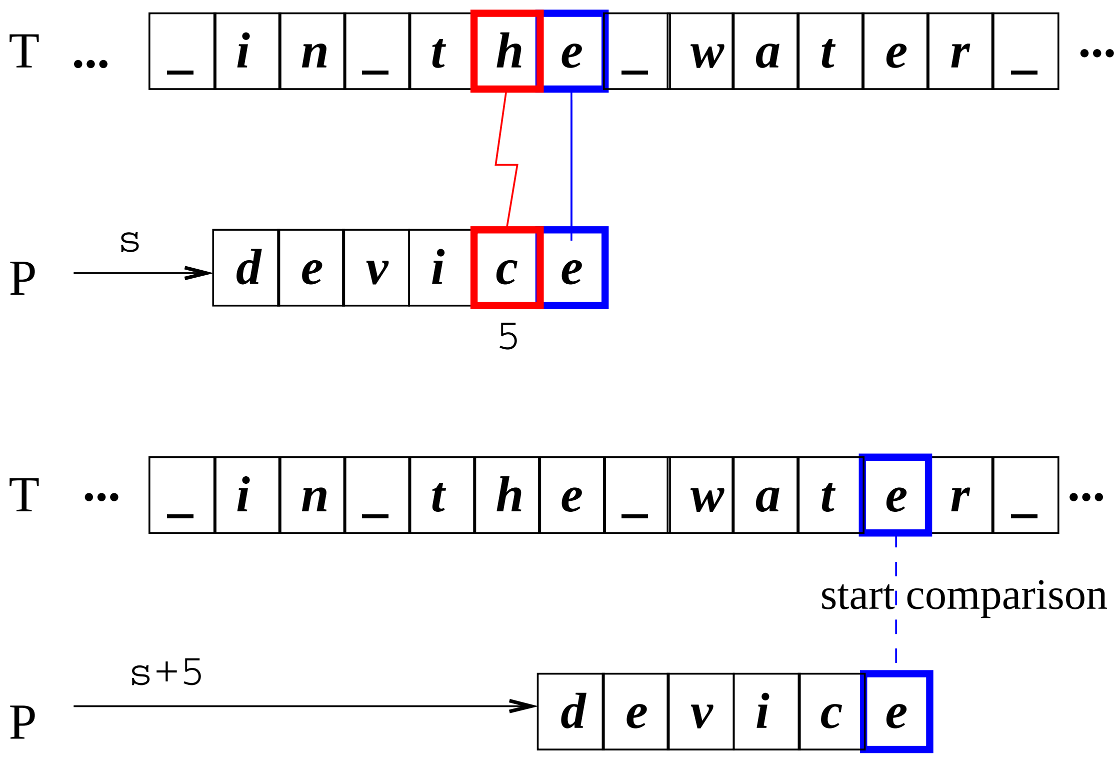
   (c) Hence, the running time of `KMP_StringMatcher` is $O(n) + O(m) = O(m + n)$.

The Boyer-Moore (BM) Algorithm

The Boyer-Moore (BM) algorithm slides P from left to right; however it compares P and T from **right to left**, i.e., $P[m]$ will first compare with $T[i]$. If they match, it then compares $P[m-1]$ with $T[i-1]$, etc. Else, it slides P to right, and compare $P[m]$ with T again.

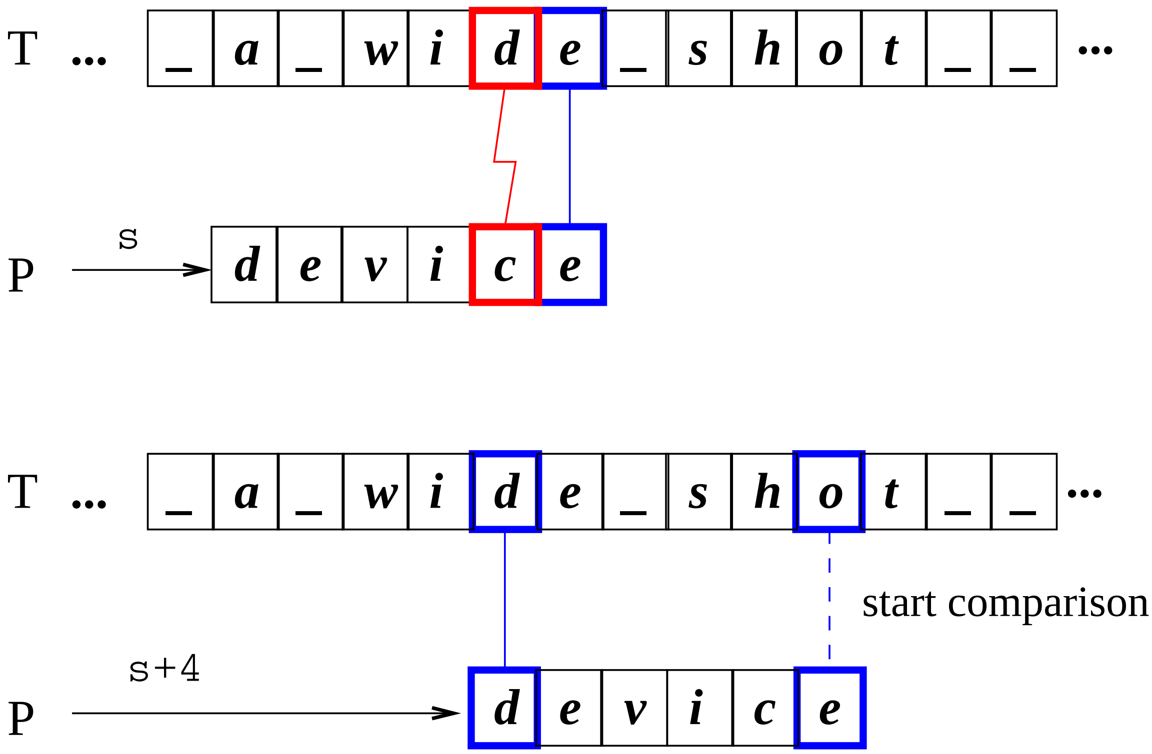
The BM Algorithm : the bad-character heuristic

One insight of BM algorithm is that, if there is a mismatch between $P[j]$ and $T[i]$, and $T[i]$ does not appear in P . P should be advanced by j.



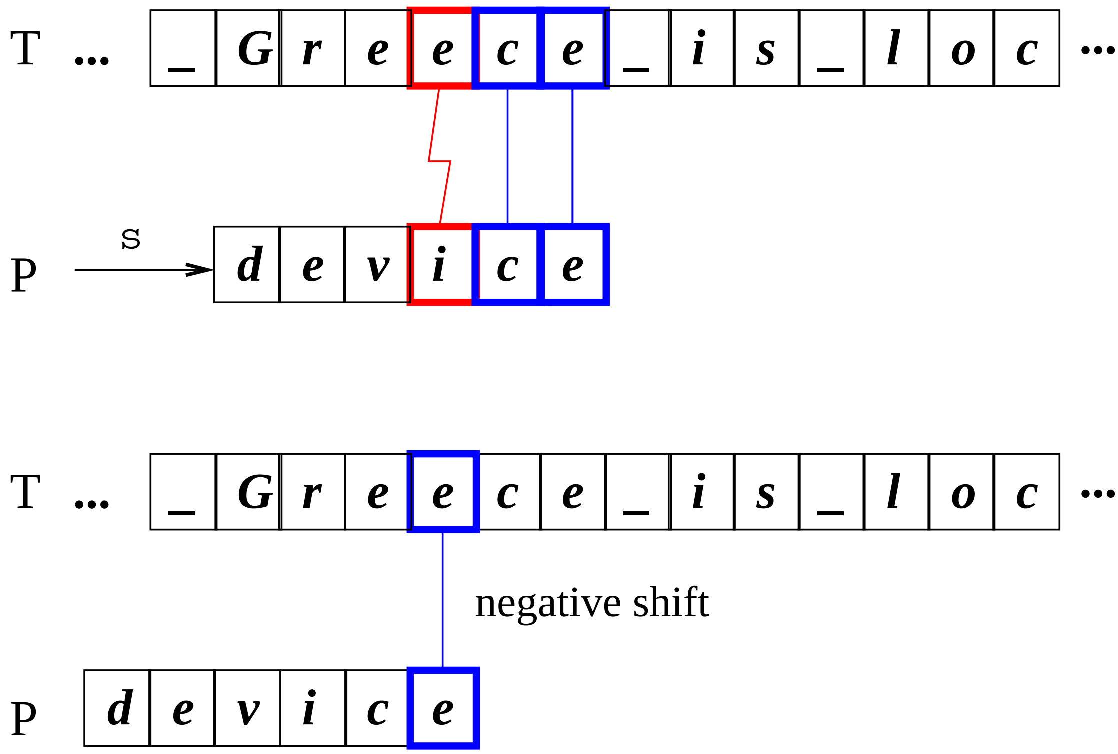
The BM Algorithm : the bad-character heuristic

If $T[i]$ appears in P , shift P such that $T[i]$ is aligned with the *rightmost* occurrence of $T[i]$ in P .



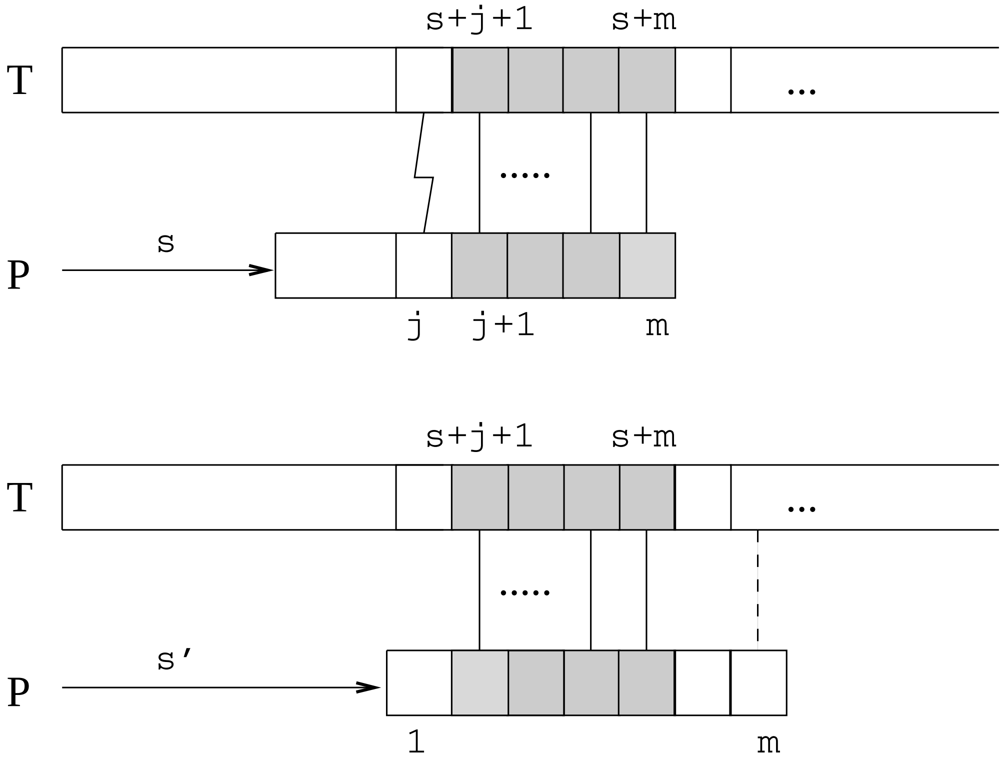
The BM Algorithm : the bad-character heuristic

If it happens the alignment of T and P gives a *negative* shift value, then just ignore it.



The BM Algorithm : the good suffix heuristic

Similar to the KMP algorithm, if the current shift is s, and it is a mismatch at $P[j]$, then we know $P[j+1..m]$ = $T[s+j+1..s+m]$. Then we can shift P by s' such that T is aligned with the *rightmost* occurrence of $P[j+1..m]$.



The BM Algorithm

The BM Algorithm takes the *larger* shift amount computed by bad-character heuristic and good-suffix heuristic.