New Techniques for Bounding the Channel Capacity of Read/Write Isolated Memory*

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A serial binary $(0, 1)$ memory is *read isolated* if no two consecutive positions in the memory may both store 1's; it is *write isolated* if no two consecutive positions in the memory can be changed during rewriting. Such restrictions have arisen in the contexts of asymmetric error-correcting ternary codes and of rewritable optical discs etc.. A *read/write isolated memory* is a binary, linearly ordered, rewritable storage medium that obeys both the read and write constraints. This type of memory was first considered by Cohn who examined its channel capacity. Let k be the size of the memory in binary symbols, r the lifetime of the memory in rewrite cycles and $N(k, r)$ the number of distinct sequences of r characters. For fixed k, the *channel capacity*, measured in bits per rewrite, is defined as $C_k = \lim_{r\to\infty} \frac{1}{r} \log_2 N(k, r)$. The channel capacity of the read/write isolated memory, in bits per symbol per rewrite, is defined to be $C = \lim_{k\to\infty} \frac{1}{k} C_k$.

Cohn originally analyzed the capacity C and proved $0.509... \le C \le 0.560297...$. This bound was recently considered by Yong et al. and improved to $0.53500... \le C \le 0.55209...$. Their approach was to model the problem as a constrained two-dimensional binary matrix problem and then modify recent techniques for dealing with *transfer matrices* of such constraints. Better and better bounds could be found by finding the eigenvalues of larger and larger transfer matrices.

In this paper we introduce new *compressed matrix* techniques. Taking advantage of recursive properties of the vertical and horizontal transfer matrices of the constraint we prove that

$$0.5350150... \le C \le 0.5396225....$$

The new contribution of this paper is to show that it is possible to take advantage of the recursive structures of the transfer matrices to (i) build other matrices of the same size whose eigenvalues yield provably better bounds or (ii) build smaller matrices whose largest eigenvalues are the same as those of the transfer matrices. Thus, it is possible to get the same bounds with less computation. We call these approaches *compressed matrix* techniques. While technique (ii) was specific to this problem technique (i) is applicable to many other two-dimensional constraint problems.

*This work was supported by Hong Kong RGC CERG grants HKUST6137/98E, HKUST6162/00E and DIMACS. *Email*: {xryong, golin}@cs.ust.hk

Proceedings of the DATA COMPRESSION CONFERENCE (DCC'02)
1068-0314/02 $17.00 © 2002 IEEE