Research Summary and Plan

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My research interests lie in areas of algebraic graph theory, combinatorics, linear algebra with applications, and analysis of algorithms. The recent publications can be accessed from http://www.math.uprm.edu/˜xryong. Below is a simple summary which is described by the topics.

1. Counting Structures in Graphs: The number of *structures*, e.g., spanning trees, independent sets, perfect matchings, Hamiltonian cycles, acyclic orientations, cycle covers, k-colorings etc., is an important quantity in graph theory and its applications can be found in different areas of mathematics, physics and chemistry. We generalized the standard ideas and techniques for counting the number of structures and showed that, for a recursive graph (a graph that can be constructed recursively by similar graphs with smaller sizes), the number of structures satisfy linear recurrence relations. We derived recurrence relations and discussed the asymptotic properties of the number of spanning trees and the number of independent sets for certain classes of such graphs. Below we confine ourself to simply talk about spanning trees.

In general obtaining the exact number of spanning trees in a graph is difficult. However, for certain classes of graphs, deriving explicit or recurrence formulas has been proven to be possible. For example, Boesch et al. and Bedrosian conjectured, independently, that if C^2 is a square cycle of n nodes, then the number of spanning trees of C^2 is nF_n^2, where $F_n = F_{n-1} + F_{n-2}$, $F_0 = 0$, $F_1 = 1$ are the *Fibonacci numbers*. This conjecture has now been given different proofs independently by D. Kleitman and B. Golden, F. Boesch and H. Prodinger, and myself and F. Zhang (where our proof is simplest). Since then we have developed *techniques* for deriving general formulas for the number of spanning trees in different classes of graphs and, in particular, we proved that the number of spanning trees of a circulant graph satisfies a linear recurrence relation. These formulas greatly reduce the amount of time needed to calculate the number of spanning trees. Most recently, we analyzed the combinatorial properties of the numbers.

2. Special Matrices: On this topic my research was focused on *pseudo-tournament matrices, nonnegative matrices, M-matrices, H-matrices, Elliptic matrices, etc.*, with special attention paid to eigenvalues, inversion, and their applications (e.g., application to the stability analysis of a dynamical system). My deepest result is the proof of a conjecture posed by R. Horn and C. Johnson in their book "Topics in Matrix Analysis, Cambridge, 1991." (*Independently*, the same conjecture was also posed by Fiedler and Markham in Fiedler's book "Special Matrices and Their Applications in Numerical Mathematics, Dordrecht & Praha, 1986." It states that: let A be an $n \times n$ nonsingular M-matrix, A^{-1} its inverse, and let $A \circ A^{-1}$ be the Hadamard product of A and A^{-1}. Then $q(A \circ A^{-1}) \geq \frac{2}{n}$, where $q(B)$ stands for the least eigenvalue (in modulus) of B. Before I proved its validity, only an incorrect proof had previously appeared in the literature and $q(A \circ A^{-1}) \geq \frac{1}{n}$ was known and used. Most recently, we obtained new properties for tournament related matrices.

3. Constrained Codes: I am also interested in Shannon theory of constrained codes. My work was focused on analyzing the *channel capacities* of two-dimensional constrained codes. The problems are closely related to certain kinds of *counting problems in combinatorics*, e.g., counting the sequences which satisfy certain restrictions, and are motivated by the problems in information theory arising from the codes for mass data storage/transmission systems. While the study of the Shannon capacity of one dimensional codes is well developed, the study of multi-dimensional codes is still in its infancy (although recently very active). In our work, we established new theoretical results on two-dimensional constrained codes and developed general techniques for bounding their capacities. Our main tools are *matrix theory, graph spectra and combinatorial analysis.* As examples, we considered and improved the analysis of the capacity of runlength limited $(1, \infty)$ two-dimensional constrained codes (previously considered by N. Calkin and H. Wilf), the channel capacity of read/write isolated memory (previously considered by M. Cohn), and the number of placements of non-attacking kings (a combinatorial problem due to D. Knuth previously considered by H. Wilf) etc.. We were also able to apply the techniques developed to attack other non information-theory related problems.

4. Spectra of Graphs: The topological structure of a graph can be analyzed by studying eigenvalues of the graph; The spectral technique can often be used to attack certain classes of counting problems in combinatorics; In applied sciences, e.g., statistical physics, quantum chemistry, etc., the eigenvalues of graphs represent energy levels of molecules. I have been interested in this topic since 1996. My deepest result can be described as follows: In the set of graphs $\{G_m\}$ of order n and with negative third largest eigenvalue, there is only one graph that has no eigenvalue equal to -1, all others have the property that there is an index $k \leq \frac{n}{2}$ such that $\lambda_j = -1,\ j = k, k+1, \cdots, n-k+1$, where λ_k is the kth largest eigenvalue of G (in some cases the j can run up to $n-k+2$).

Short Term Research Plan:

There are many interesting eigenvalues related combinatorial problems (for example, the free energy of Ising model, the ice-type model, the multi-dimensional dimer problem) in discrete applied mathematics. Combining the techniques (e.g, spectral technique, transfer matrix technique. etc.) from matrix analysis, algebraic graph theory and combinatorics turns out to be efficient in attacking many such problems and recently, there has been active research utilizing the techniques.

In the coming years, I plan on pushing my interest further to work on eigenvalues related combinatorial problems. I also plan on developing new techniques (e.g, compressed matrix technique) for analyzing the asymptotics of the number of combinatorial structures.